DOCUMENT RESUME

ED 339 590 SE 052 237

AUTHOR	Sowder, Judith; And Others
TITLE	Understanding as a Basis for Teaching: Mathematics and Science for Prospective Middle School Teachers. Final Report.
INSTITUTION	San Diego State Univ., CA. Center for Research in Mathematics and Science Education.
SPONS AGENCY	National Science Foundation, Washington, D.C.
PUB DATE	May 91
CONTRACT	NSF-TPE-50315
NOTE	348p.
PUB TYPE	Reports - Descriptive (141) -- Tests/Evaluation Instruments (160)
EDRS PRICE	MF01/PC14 Plus Postage.
DESCRIPTORS	Attitude Change; Attitude Measures; *Biology; *Calculus; *Cognitive Development; Cognitive Measurement; Concept Formation; *Curriculum Design; Higher Education; Instructional Design; Intermediate Grades; Junior High Schools; Mathematics Achievement; Mathematics Curriculum; Mathematics Education; Mathematics Instruction; Middle Schools; Preservice Teacher Education; Probability; Rational Numbers; Science Curriculum; Science Education; Science Instruction; Statistics; *Student Attitudes; Teacher Education; *Teacher Education Curriculum
IDENTIFIERS	*Constructivism

ABSTRACT

When teachers possess detailed knowledge about children's thinking and problem solving, it can profoundly affect their knowledge of their students and their planning for instruction. Reported is a project designed to demonstrate the feasibility of redesigning courses for prospective teachers in mathematics and science by incorporating into the courses research results from cognitive science and by focusing on the development of pedagogical content knowledge. After the background of the project is presented, the section on the project's implementation describes the preparation stages and the three courses offered in four sections. Section 1: Cognitive Seminar for Teacher Preparation Project, describes the organization of knowledge, how it will be presented in the classroom, and how student learning and attitudes towards learning will be assessed for the three courses being designed. Section 2 describes the mathematics course entitled "Calculus for Middle School Teachers," including course development activities, evaluation data, and implications for curriculum development. Section 3 describes the mathematics course entitled "Mathematics Course for Elementary/Middle School Teachers: Rational Numbers, Proportional Reasoning, Probability, Statistics," including course planning, instruction, information about students, evaluation of student affect and knowledge measures, and a discussion of instructor impressions of students. Section 4 describes the biology course entitled "Process and Inquiry in Life Science," including an overview of the course, the materials and methods used during teaching, results of science process skills and affective attitudes of experimental and comparison groups, and discussion and conclusions from the results. Appendices including pertinent documents with respect to activities used in lessons taught, evaluation instruments for knowledge and attitude measures, data gathered in the study, and reports made at the Psychology of Mathematics Education are given. (MDH)

ED339590

UNDERSTANDING AS A BASIS FOR TEACHING:

MATHEMATICS AND SCIENCE FOR PROSPECTIVE MIDDLE SCHOOL TEACHERS

FUNDED BY NATIONAL SCIENCE FOUNDATION
AS TPE 8950315

FINAL REPORT

Principal Investigators:
Judith Sowder, Sandra Marshall, and Cheryl Mason
Additional Investigators:
Nadine Bezuk, Kathleen Fisher, Alfinio Flores, Douglas McLeod,
Stephen Reed, Phoebe Roeder, Larry Sowder

May 1991

The preparation of this report was supported by the National Science Foundation, Grant No. TPE 50315. The opinions expressed here do not necessarily reflect the position, policy, or endorsement of the National Science Foundation.

SE 052 237

U.S. DEPARTMENT OF EDUCATION
Office of Educational Research and Improvement
EDUCATIONAL RESOURCES INFORMATION
CENTER (ERIC)
☒ This document has been reproduced as received from the person or organization originating it.
☐ Minor changes have been made to improve reproduction quality.
- Points of view or opinions stated in this document do not necessarily represent official OERI position or policy.

"PERMISSION TO REPRODUCE THIS
MATERIAL HAS BEEN GRANTED BY
Judith Sowder

TO THE EDUCATIONAL RESOURCES
INFORMATION CENTER (ERIC)."

BEST COPY AVAILABLE

TABLE OF CONTENTS

BACKGROUND OF THE PROJECT

Purpose 1
Incorporating Research Results from Cognitive Science 1
The Development of Pedagogical Content Knowledge 2
Plan 2

IMPLEMENTATION OF THE PROJECT

COGNITIVE SEMINAR FOR TEACHER PREPARATION PROJECT 5
  Summary 5
  Organization 5
  Representation 6
  Assessment 7

A. CALCULUS FOR MIDDLE SCHOOL TEACHERS (Math 121) 8
  Course Development Activities 8
  Evaluation Data 9
  Implications for Curriculum Development 13
  References 14

B. MATHEMATICS COURSE FOR ELEMENTARY/MIDDLE SCHOOL TEACHERS: RATIONAL NUMBERS, PROPORTIONAL REASONING, PROBABILITY, STATISTICS (Math 211) 15
  Course Planning 15
  Instruction 16
    Lessons on Rational Number 16
    Lessons on Proportional Reasoning 17
    Lessons on Statistics 17
    Lessons on Probability 17
  Information about Students 18
  Evaluation 18
    Affect measures 18
      Results from Attitudinal and Belief Measures 18
      Results from Final Questionnaire on Learner Attitudes 22
    Knowledge measures 24
      Rational Number Concepts 24
      Meanings of the Operations 28
      Proportional Reasoning 30
      Statistics 32
      Probability 33
  Discussion 35
    Instructor Impressions of Students 35
  References 35

C. PROCESS AND INQUIRY IN LIFE SCIENCE (NS 412C) 37
Overview 37
Background 37
Course Planning 37
Cognitive Strategies 37
Content Competencies 38
Affective Attitudes 39
Science Process Skills 39
Materials and Methods 39
Results 40
Addressing Naive Conceptions (Experimental Group) 41
Science Process Skills (Experimental and Comparison Groups) 42
Affective Attitudes (Experimental Group) 44
Quantitative Attitude Measures 44
Qualitative Attitude Measures 46
Learning Experiences and Strategies 46
Content Learning 49
Process Skills 50
Negative Response to Hands-on Science 52
End of Course Reflections 52
Discussion and Conclusions 54
Positive Outcomes 54
Negative Outcomes 54
Materials and Strategies Developed 55
Instructional Design 55
Strategies 55
Instruments 56
Key Challenges for the Future 56
References 57

Publications and Presentations Acknowledging this Project 58

Tables

A.1: Background Data on Participating Students 10
A.2: Pretest and Posttest Mean Scores 11
A.3: Interview Questions 12

C.1: Cognitive Strategies 38
C.2: Content Competencies 38
C.3: Pre- and Post-Performance on Biology Content Questions 41
C.4: Pre- and Post-Performance on Items Pertaining to the Urinary System and Plant Respiration 42
C.5: Process Skills 43
C.6: Example of Pre- and Post-Attitude Survey Items 45

Appendices

A.1 Math 121: Calculators in Calculus: That's the Limit
A.2 Math 121: Linear Functions
A.3 Math 121: Evaluation Instruments

B.1 Math 211: Chapter: "Using Principles from Cognitive Psychology..."
B.2 Math 211: Poster Session for PME
B.3 Math 211: Group Activities, Assignments, Quizzes, Tests
B.4 Math 211: Examinations
B.5 Math 211: Questionnaire on Learner Goals/Theories of Intelligence and Data
B.6 Math 211: Final Questionnaire on Learner Attitudes and Data
B.7 Math 211: Rational Number Questionnaire and Data
B.8 Math 211: Interview Forms and Data

C.1 NS412C: Demographics and Attitude Presurveys
C.2 NS412C: Content and Process Skills Pretest
C.3 NS412C: Sample Lessons
C.4 NS412C: Attitude Postsurvey
C.5 NS412C: Final Examination

BACKGROUND OF THE PROJECT

Purpose

The purpose of this project was to demonstrate the feasibility of redesigning courses in mathematics and science by incorporating into the courses research results from cognitive science and by focusing on the development of pedagogical content knowledge. The three courses we selected for this project are all courses in our program for preparing middle school teachers of mathematics and science. In mathematics we redesigned a one-semester course in calculus and a one-semester course in which the primary foci are rational number understanding, probability, and statistics. In science we redesigned a one-semester course in biology.

Incorporating Research Results from Cognitive Science

In recent years, cognitive psychological research has contributed to our understanding of the learning process. Much of this research has been done in the areas of mathematics and science learning. Linn (1986) has pointed out that science (and mathematics) education would be strengthened by building upon what we already know about the cognitive structure of the subject matter. The work of Carpenter, Fennema, and Peterson (Carpenter, Peterson, Fennema, Chiang, & Loef, 1989) has shown that when teachers possess detailed knowledge about children's thinking and problem solving, it can profoundly affect their knowledge of their students and their planning for instruction.

One frequent theme of this research is the view of the learner as the active constructor of his or her own knowledge. A number of studies document how individuals organize the knowledge they acquire and how that organization changes with time and experience (excellent summaries are given by Resnick, 1986, and Cobb, 1987). A particularly interesting aspect of this research is the study of student preconceptions and misconceptions in mathematics and science (see references under description of seminar). Not until the teacher is aware of the student conceptions that interfere with a full understanding of fundamental ideas in mathematics and science, can he/she go about creating the instructional conditions that change and transform those conceptions. This fact was recently acknowledged in *Everybody Counts*, a document on the future of mathematics education published by the National Research Council (1989): "Clear presentations by themselves are inadequate to replace existing misconceptions with correct ideas. What students have constructed for themselves, however inadequate it may be, is often too deeply ingrained to be dislodged with a lecture followed by a few exercises" (p. 60). The time is ripe to incorporate research findings from cognitive psychology into teacher preparation (Carpenter et al., 1989). Shulman (1987) has pointed out that such research-based knowledge is at the very heart of his definition of needed pedagogical content knowledge.

The Development of Pedagogical Content Knowledge

Pedagogical content knowledge refers to the representation and formulation of subject matter to make it comprehensible to the learner. Pedagogical content knowledge requires a solid foundation in the content areas in which one teaches, then goes beyond to blend content and pedagogy through the examination of the structure, major organizing concepts, and important ideas and skills of the content areas. It is a blend of content and pedagogy and demands an understanding of the content in terms of how it can be organized and represented for instruction (Shulman, 1987). This view of teaching requires that teachers have a deep understanding of both the content and the processes of learning. "Those who would teach mathematics need to learn contemporary mathematics appropriate to the grades they will teach, in a style consistent with the ways in which they will be expected to teach" (National Research Council, 1989, p. 63). In this project, we attempted to integrate these two aspects of learning through our incorporation of cognitive science principles and methodology into our existing coursework.

Plan

This was a one-year project. During the fall term, all participating faculty were involved in a preparatory seminar focusing on instructional implications of research in cognitive science, and groups of faculty planned the content modifications and instructional delivery of each of the three pilot courses. During the spring term, faculty pairs team-taught the courses and carried out the evaluation.

References

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499-531.

Cobb, P. (1987). Information-processing psychology and mathematics education -- A constructivist perspective. *The Journal of Mathematical Behavior, 6*(1), 3-40.

Linn, M. C. (1986). *Establishing a research base for science education: Challenges, trends, and recommendations* (Report of a National Conference). Berkeley, CA: Lawrence Hall of Science.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education.* Washington, DC: National Academy Press.

Resnick, L. B. (1986). The development of mathematical intuition. In M. Perlmutter (Ed.), *Perspectives on intellectual development: The Minnesota Symposia on Child Psychology* (Vol 19, pp. 159-194). Hillsdale, NJ: Erlbaum.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

IMPLEMENTATION OF THE PROJECT

COGNITIVE SEMINAR FOR TEACHER PREPARATION PROJECT

Sandra Marshall and Stephen Reed

Summary

During the fall semester, the faculty associated with the project met regularly to discuss cognitive issues and to plan cognitive interventions or strategies for use in the developing courses. These meetings were conducted as informal seminars led by two cognitive psychologists. The first three meetings of the seminar were devoted to examinations of current research issues in the relevant subject areas, followed by content descriptions of the courses that were to be created or modified. A continual emphasis of the seminar was the direct and practical implication of cognitive research findings in SDSU classrooms, particularly in Math 211, Natural Science 412C, and Math 121 courses.

Three general themes dominated the discussions: organization, representation, and assessment. They are described briefly below.

Organization

Organization of knowledge was the predominant focus of the seminar. Two instructional questions were addressed with the help of cognitive science research: (1) How can information be organized to help students acquire a cohesive knowledge structure? (2) How should the information be presented? The seminar discussions examined both of these issues with respect to the target courses.

An issue that received a great deal of attention in the discussions was how to take advantage of top-down and bottom-up information processing. Very often the teacher, operating under a bottom-up framework, specifies the necessary facts and procedures that students should acquire through bottom-up processing. These facts and procedures become the core of the instruction. What is missing is the presentation of the top-down framework for the students, so that they, too, may see the overall picture and begin to organize their knowledge accordingly. Students frequently learn isolated bits and pieces of information that bear little relation to each other. Their knowledge of the subject matter is fragmented and lacks cohesion. Thus, an essential ingredient of the target courses was an organization that helped students relate the knowledge they were acquiring to other knowledge they had.

One approach to the organizational materials centers on identifying primary schemas that reflect competence in the subject matter. From a problem-solving perspective, schemas are the knowledge structures that allow an individual to recognize and deal with various experiences and situations in the subject area. Learning is characterized as the development of appropriate schemas, where the broad framework of the schema is developed, and necessary related facts and techniques are then linked directly to this framework. Access and retrieval of information from memory results in the activation of the entire framework rather than a single piece of information.

An instructional technique for improving students' organization of concepts is to teach them to construct semantic networks. Semantic networks consist of concepts connected by links that show the relation between concepts. Research has shown that construction of networks can improve students' performance on essay and short-answer tests that assess the organization of knowledge. The SemNet software, which allows for the construction of large semantic networks on a computer, was used in some pilot courses to evaluate and improve students' organization of concepts.

Associated with the issue of how knowledge is organized in students' memory is the finding that students tend to retain new information best when it builds upon their own previous knowledge. The direct implication for instruction is that new material should be clearly and explicitly linked to understandings already demonstrated by the students. Two general results of the discussion on organization were: (1) that the scope of the courses should be curtailed (in terms of the number of individual concepts to be addressed) to promote depth rather than breadth in students' learning; and (2) that frequent student misconceptions or naive conceptions should be addressed directly in such a way that students confront them in contexts that are familiar and understandable.

Representation

A second general theme of the seminar was the nature and importance of representing information, especially through visual images. A number of cognitive science studies have shown that students recognize and use information from pictorial or graphic presentation more easily than from verbal presentation. Several modes of representation were discussed, including graphs, diagrams, and models.

A second issue was students' use of analogies as they learn. Several recent cognitive science studies have pointed to the importance of examples for students. The need to develop relevant and useful examples was explored for each of the proposed classes. Related to this issue is the notion of transfer of skills and understanding. Several recent studies in problem solving were discussed, with particular attention to the need to establish direct links between established examples and new problems or situations.

There are two kinds of links that should facilitate the effective use of analogies. First, students need to recognize that problems can be structurally identical (isomorphic) when the story content changes. There is abundant evidence that an important aspect of acquiring expertise is the ability to select analogies on the basis of structural similarities rather than on similarity of story content. Second, problems that have similar story content can have different solutions. In this case, the problem solver must learn how to modify the solution of an analogy through learning additional procedural knowledge or using multiple analogies.

Assessment

The final theme was the assessment of student learning that moves beyond traditional boundaries. Of particular importance were assessment techniques for looking at the development of understanding of broad concepts and the reduction of naive conceptions or misconceptions. A departure from standard assessment procedures is the use of pretests to identify students' misconceptions at the beginning of the course. Tests given during the course could then be used to monitor the effectiveness of instruction in changing initial misconceptions.

Novel methods of assessment, such as the previously mentioned construction of semantic networks, were discussed along with assessment issues involved in evaluating students as they worked in group activities. Techniques such as open-ended interviews, the keeping of journals, and videotaping were considered as well as possible ways of scoring them.

Another aspect of assessment is self-evaluation. Ideally, students should be able to evaluate how well they understand the material. However, recent work in psychology has shown that self-evaluation is often poor; that students can not accurately predict how well they will perform on a test. We therefore need to improve their awareness of their progress in a course so that they can devote further attention to those content areas in which they are deficient.

CALCULUS FOR MIDDLE SCHOOL TEACHERS: Math 121

Alfinio Flores and Douglas McLeod

Course Development Activities

Our goal was to provide a "lean and lively" introduction to the major ideas of calculus (Douglas, 1986), an introduction that would be compatible with the current efforts at reform in mathematics education, as well as accessible to students with a limited mathematical background. The overall metaphor for our thinking comes from current efforts to make calculus courses a pump, not a filter (Steen, 1988), efforts that focus on developing courses with a strong conceptual emphasis and minimal prerequisites in algebraic computation. We also integrated the use of graphing calculators and computer software into the course. Each student checked out a graphing calculator (the Casio fx-7000g) for the semester (and each returned it in good condition at the end of the term!).

The general outline of the course was to develop the notion of limit, use it as the foundation for the concepts of differentiation and integration, and then to link these two major ideas through the unifying power of the Fundamental Theorem of Calculus. The development of the limit concept was accomplished through a unit on limits of sequences that was based on the use of scientific calculators. See Appendix A.1 for "Calculators in Calculus: That's the Limit" for details on this unit. This work on limits was followed by an introduction to linear functions, slope, and applications; these ideas formed the foundation for the introduction of the derivative. For a sample of the related instructional activities, see the material on "Linear Functions" in Appendix A.2.

Our introduction of the derivative concept was based initially on the idea of velocity; this section of the course made heavy use of the classic presentation by Sawyer (1975), especially chapters 2-5. Once the ideas were introduced, various applications of the notion of derivative were discussed; the text for the course (Goldstein, Lay, & Schneider, 1987) was the primary reference for these applications, and the materials were adapted so that the use of graphing calculators would be appropriate. The substantial power of the graphing calculator enabled students to use trigonometric and exponential functions, as well as polynomial and rational functions, in solving various kinds of problems dealing with rates of change, maximum and minimum points, and other traditional topics from the calculus curriculum.

The notion of integral was introduced as the area under a curve, and then the relationship between the integral and the anti-derivative was investigated. A major resource for this section of the course was the classic work of Courant and Robbins (1978).

This outline of the course content was influenced by a number of theoretical considerations based mainly on research in cognitive science. For example, we tried to build up abstract concepts out of more concrete examples, such as using velocity as the introduction to derivative. We also used analogical reasoning, such as talking about the seasons of the year as periodic, and then connecting that idea to the periodicity of trigonometric functions. We tried to make clear to students that our goal was to improve their ability to think conceptually about the ideas of calculus, not just to work on computational proficiency. We attempted on a regular basis to develop strong connections among important ideas, striving to build a semantic network that would enable students to see calculus as an integrated collection of important ideas, rather than as a disconnected set of procedural skills. Our evaluation activities were designed to help us determine our successes and failures in these efforts.

Evaluation Data

The evaluation of the course used questionnaires, paper-and-pencil assessment of student performance (in Appendix A.3), and interviews to gather cognitive and affective data that were related to student achievement. Cognitive assessment focused mainly on higher-order thinking skills in the domain of calculus. Data from the experimental class were compared with data from a traditional calculus class that was taught during the same semester.

Students in the experimental section were recruited from the ranks of prospective elementary school teachers. They were invited to enroll in a special section of Math 121, Calculus for the Life Sciences. There were 12 students who enrolled in the course; one of those dropped in mid-semester due to a family crisis, and one other quit attending at the end of the term with no explanation. Of the ten students who completed the class, eight consented to be interviewed.

A "control" group of 11 students was selected at random from another section of Math 121. One of those students dropped out of class without taking the final exam; six of the students who completed the course agreed to be interviewed.

At the beginning of the semester students filled out a background questionnaire and took a pretest . Data from this assessment indicated that the students had similar mathematics backgrounds, although their majors were different. As Table 1 indicates, seven of the ten students who completed the experimental section were elementary education majors, and eight of the students in the control class were majoring in the life sciences.

Table 1

Background Data on Participating Students

Characteristic	Experimental	Control
Major		
Elementary Teaching	7	0
Life Science	1	8
Other	2	2
Mathematics Background		
College Algebra	5	4
Intermed. Algebra	5	6

Data on the students' performance on the pretest is summarized in Table 2. Scores were quite low on the six-point pretest, which dealt with linear and quadratic functions and their graphs. Although both classes did poorly, the experimental class got twice as many problems correct as did the sample from the control class.

The final exams for the two classes were different, since the classes had covered different material. For example, the standard curriculum for Math 121 deals only with the derivative and its applications, and the integral is not introduced until the second semester of the course. However, there were seven items on the final that were common to both final exams. These items dealt with graphing a cubic function, solving a traditional max/min problem, and describing the flight of a helicopter when the distance function is given. The results (again in Table 2) suggest that the students in the experimental section were correct on about twice as many items as the control students. Of course, one must keep in mind that although both groups were allowed to use calculators on the final, only the experimental group was provided with the more powerful graphing calculators. The data from these two paper-and-pencil assessments were supplemented by interviews with students from both the experimental and control sections.

Table 2
Pretest and Posttest Mean Scores

Assessment	Experimental	Control
Pretest	2.0 (n = 9)	0.8 (n = 10)
Posttest		
Graphing	1.7	1.1
Max/Min	1.1	0.1
Distance	1.0	0.3
Total	3.8 (n = 10)	1.5 (n = 9)

Interviews were conducted with eight of the students in the experimental group and with six of the students in the control group. Although all of the students appeared willing to be interviewed, the students' busy schedules at the end of the semester made it difficult to arrange times that were convenient for the students. All interviews were tape recorded; in addition, the interviewer kept notes on each session. The questions asked in the interview are listed in Table 3. When necessary, these questions were followed with requests for clarification. If the students had difficulty understanding the question, the interviewer rephrased the question to be more specific. The interviews with the students in the experimental class were conducted during the week before their final exam; the students in the control class were interviewed the week after their final exam.

Students in both classes generally reported positive changes in beliefs and attitudes toward mathematics as a result of this course. Students in the control class were slightly more likely to think of mathematics in terms of following rules, and students in the experimental class were slightly more likely to report that they liked math more after this class. Students in both classes made use of technology, but students in the control class were generally limited to finding logs or doing arithmetic on a standard scientific calculator of their own. All students in the experimental section reported extensive use of their calculators in graphing activities, and about half also used the computer software graphing package (Master Grapher) that was made available to them. Students reported no serious difficulty in using these technological aids, even though it took the experimental group some time to get used to having to adjust the "window" of the calculator in order to find a proper graph of some functions. Students in the experimental group were uniformly positive about the use of graphing calculators, just as the control group was positive about using their scientific calculators. One student in the control group, however, was concerned that using the calculator might encourage students to "skip steps" and to "forget the quadratic formula."

Table 3
Interview Questions

Has Math 121 influenced your beliefs about mathematics?

Has Math 121 influenced your attitudes toward mathematics?

Did you use calculators or computers in Math 121? If so, how did they influence your performance? Were there any difficulties in using them?

What is a function?

What is a limit?

What is a derivative?

What is an integral? (Only for students in the experimental class.)

What are the important relationships among these "big ideas" of calculus?

All students had difficulty describing clearly the important concepts of calculus. Students in the experimental class were more likely to describe functions as a relationship or correspondence, and students in the control group were better able to give examples of applications of functions (primarily biological, reflecting the majority of majors in that field). About half of the experimental group could say something about the idea of limit (a number that you get closer and closer to, reducing the error of approximation), but their explanations were always vague and uncertain. Students in the control class were even less able to express ideas about limits clearly. In both classes students would claim that they could find the limit, given a problem, but that the idea was too hazy for them to express in words.

Most students in both classes talked about the derivative in terms of its applications (e.g., finding maximum and minimum points) rather than in terms of its definition. A smaller fraction of the students could describe the derivative as a way of finding the slope or the rate of change of the function. It was surprising that none of the experimental students mentioned velocity as an interpretation of the derivative, since that was the example that we used to generate the concept.

The concept of the integral was covered only in the experimental section and it was the subject being discussed in class at the time of the interviews. As a result, most students discussed the integral in terms of the area under the graph of the function, the only interpretation that they had seen presented up to that time. Most of the experimental group was also able to relate the integral and the derivative in terms of the fundamental theorem of calculus, but almost no students in either class could describe any connection between the concept of limit and the concept of derivative.

Implications for Curriculum Development

Students in the experimental class developed a reasonable ability to apply the notions of calculus in solving problems, and they showed some conceptual understanding of the major concepts of the field. The use of the graphing calculator appeared to be very helpful in assisting these students (who were weak in algebra skills) in developing some proficiency in calculus concepts. It appears to be relatively easy to help students develop the skills they need to use the graphing calculator to solve problems that would otherwise remain beyond their level of algebraic competence. The students had a great deal of difficulty, however, in expressing themselves clearly regarding calculus concepts. More experience in writing about these concepts or in explaining these concepts to their fellow students (perhaps in small group work) could help improve student performance in this area.

It was a bit surprising that no student brought up the notion of velocity to describe the concept of derivative. Generally the research suggests that the initial example of a concept tends to be quite powerful and remains an important influence on the students' thinking for some time. In this case, however, it appeared that this initial example of the concept was overtaken by the more powerful and general notion of rate of change, or the slope of the function. This unexpected result could be due in part to the extensive use of the graphing calculator; the slope of the graph became a very salient part of the students' study of functions, something that they could see. As one student put it, the calculator really allowed her to visualize what was happening to the function. Velocity, of course, is not easy to visualize.

There was one example, however, of how students' initial introduction to a concept "hangs on," even when it is not useful and not even a good introduction. Two students (one experimental, one control) described a function as a graph that only crosses the x-axis once (!), a corruption of a procedural test of the distinction between the graph of a relation and the graph of a function. This test was not discussed in this course, so clearly it was being recalled from earlier classes in pre-calculus mathematics. It would be helpful if such perfidious procedural confusion could be eliminated from the pre-calculus curriculum.

In summary, the experimental course was reasonably successful in using technology to help students with inadequate algebra knowledge as they tried to understand all the major concepts of calculus in one semester. We need to find new ways to encourage students to concentrate on developing the conceptual knowledge that is most important for prospective teachers of school mathematics.

References

Courant, R., & Robbins, H. (1978). *What is mathematics?* Oxford: Oxford University Press.

Douglas, R. G. (Ed.). (1986). *Toward a lean and lively calculus*. MAA Notes No. 6. Washington, DC: Mathematical Association of America.

Goldstein, L. J., Lay, D. C., & Schneider, D. I. (1987). Calculus and its applications. (4th ed.) Englewood, NJ: Prentice-Hall.

Sawyer, W. W. (1975). *What is calculus about?* Washington, DC: Mathematical Association of America.

Steen, L. A. (Ed.). (1988). *Calculus for a new century: A pump, not a filter*. MAA Notes No. 8. Washington, DC: Mathematical Association of America.

MATHEMATICS COURSE FOR ELEMENTARY/MIDDLE SCHOOL TEACHERS: RATIONAL NUMBERS, PROPORTIONAL REASONING, PROBABILITY, STATISTICS (Math 211)

Larry Sowder, Judith Sowder, and Nadine Bezuk

Course Planning

During the fall term of 1989, the three faculty met on alternate Wednesday afternoons, and on the other Wednesday afternoons we participated in a seminar attended by all the faculty involved in the project and led by our two cognitive psychologists, Sandra Marshall and Steve Reed. (That seminar series is discussed in an earlier part of this report.)

We began by reviewing and discussing the research on rational number learning, proportional reasoning, and misconceptions that interfered with learning probability and statistics. Curriculum efforts in these areas (e.g., the Quantitative Literacy Project, the Middle Grades Mathematics Project) were also reviewed. We also discussed the implications of the on-going research seminar for our course planning. The research we felt to be most relevant to our planning is described in two reports, included in Appendices B.1 and B.2.

The planning for the content and sequence of lessons was greatly influenced by this research. For example, we decided to devote much less time than usual to learning and justifying algorithms for operating on rational numbers and more time on representing, comparing, and ordering rational numbers, estimating with rational numbers, and on understanding of the meaning and effects of the operations on rational numbers. We wanted to focus on number and symbol sense. In our statistics lessons, as another example, we decided to delete instruction on standard deviation and instead to discuss average deviation since it is more readily comprehensible, leading us to believe it would give students a deeper understanding of variance from the mean.

We then planned a series of individual lessons based on the research findings and critiqued them within our group. The final versions of these lessons were later incorporated into some of the lessons used in the class.

Course planning continued throughout the instructional period. The two course instructors, L. and J. Sowder, met frequently to assess progress and to discuss plans for lessons. Although one instructor was designated as responsible for planning each lesson, the other instructor was usually in the classroom observing and assisting with group work.

Instruction

Instruction is used here in the broad sense of providing the structure and setting for learning to occur. It includes planning and leading class discussions, planning for group work, guiding and supervising group work during class, occasionally lecturing and demonstrating, and planning and carrying out assessment of learning, of attitudes, and of the instructional process.

Although the semester is 15 weeks long, three hours of class per week, this report involves only about 12 weeks of class time. During the first and last weeks of class we were interviewing students. Although the interviewing took place outside of class time, we did not wish class activities during that time to influence student answers. In addition, we were both out of town for one week. A unit on measurement and two lessons on LOGO filled out this remaining time. For the twelve week portion of the course, approximately five weeks were devoted to instruction on rational number, about two weeks on proportional reasoning (including ratio and percent), about four weeks on probability and statistics, and one week to in-class testing.

The lessons are summarized below. *Lesson* is used here as a designation for a focused collection of activities, and some lessons required several days to complete. Samples of the Group Activities referred to here can be found in Appendix B.3, together with problems assigned to be worked outside of class.

Lessons on Rational Number

Part I in particular is based on research on the different interpretations for rational number.

Part I: Interpretations and models for fractions; fractions representing quantities; establishing referents for comparing fractions; equivalent fractions.

Lesson 1: Focus on fractions as quotients, as decimal numbers; betweenness and density; introduce Explorer calculators; see Group Activity #1.

Lesson 2: Number sense, use of benchmarks, comparing and ordering, estimating sums and differences, some comparing and estimating with decimal numbers and percents. Use fraction circles and Group Activity #2.

Lesson 3: Models for fractions; number line; part-whole with area and with discrete objects; representing and ordering. Use fraction tiles, counters, calculators, Group Activity #3. Take-home quiz.

Part 2: Operations on rational numbers.

Lesson 4: Multiplication and division of fractions; meaning, development of algorithms. Use pattern blocks and Group Activity #4.

Lesson 5: Addition and subtraction of fractions; meaning, development of algorithms. Use pattern blocks, calculator, and Group Activity #5.

Lesson 6: Fractions as decimals, place value; algorithms; properties. Demonstrate with base ten materials; class discussion.

Lesson 7: Exploration of fraction operations via the Explorer with Group Activity #6.

Lesson 8. Operations on decimal numbers. Demonstration of using base ten blocks, and Group Activity #7.

Lessons on Proportional Reasoning

Lesson 1. Proportional reasoning in different types of situations. Use pattern blocks, templates for students.

Lesson 2. Ways in which variables can be related. Use handout to guide discussion.

Lesson 3. Applications of ratio and proportion. Lecture and discussion.

Lesson 4. Ratios; are ratios fractions? Discussion.

Lessons on Statistics

(Note: Some of the tables used in these lessons are from the Quantitative Literacy Project, and do not appear here because of copyright laws.)

Lesson 1. Stem-and-Leaf Plots. Use transparencies of data on children's books, building heights, thunderstorms, from Quantitative Literacy project. Demonstration followed by group work.

Lesson 2. Measures of Central Tendency; Box Plots. Use demographic information from the class for group assignment. Note: This lesson required about three days to complete.

Lesson 3. Introduction to variance; finding average deviations. Lecture, demonstration, discussion.

Lesson 4. Scatter plots and robust correlation lines. Lecture, demonstration, discussion.

Lessons on Probability

Lesson 1. Experimental and theoretical probability. Lecture on vocabulary; pair-generated data on how styrofoam cups land when tossed.

Lesson 2. Exploration of sums with two dice. Group data on M. Burns' experiment, which sum, of 2-12, will "win". (See handout in Appendix B.3.)

Lesson 3. Relationships among probabilities of events. Follow-up of questions on handout from Lesson 2, with emphasis on P(event) + P(complement), P(A or B); lecture and discussion.

Lesson 4. More complex experiments; fundamental counting principle and tree diagrams. Lecture and discussion.

Lesson 5. Binomial experiments and Pascal's triangle. Lecture and discussion.

Lesson 6. Probability concepts, and a simulation. Discussion of handout on probability concepts (See Appendix B.3); results of a simulation of 10 births (via coin tosses).

Lesson 7. Complex experiments. Discussion of problems on handout. (See Appendix B.3).

Information about Students

Twenty-one students competed the course. Although we had planned to have only students who intended to teach in the middle grades, in reality a registration mix-up caused the course to be populated with students who were late registering and were unable to schedule one of the regular classes. The students had all completed one content course in mathematics for elementary teachers. That course focused on the whole number system and on geometry. Twelve of the students had had three years of high school mathematics, one had had only one year, and the others had two years.

Evaluation

Affect Measures

Two questionnaires were given to the students. The first, administered during the first week of classes, was a measure of beliefs, attitudes, learner goals (Elliot & Dweck, 1988), and theories of intelligence (cf. Dweck & Leggett, 1988). (Note: see manuscript in Appendix B.1, pp. 22-23, for information on learner goals and theories of intelligence.) The second questionnaire consisted of questions about learning from the course, and was completed by students between the final class and the final examination for the course.

Results from attitudinal and belief measures

The attitudes and beliefs of students no doubt shape their approaches to mathematics learning and their concepts of mathematics teaching. Accordingly, we gathered data on some selected dimensions of attitudes and beliefs.

"Theory of intelligence." An interesting theory (Dweck, 1987) holds that people may view intelligence ("smartness") in two ways, perhaps only implicitly. One view is that intelligence is changeable, with the corollary that hard work and persistence can improve one's talents. The other view is that intelligence is fixed, that "you either have it or you don't," with the corollary that hard work and persistence will not really help performance much. There are obvious implications for preservice teachers who are taking a mathematics course. Many such students are quite capable, but not infrequently they have a spotty history in mathematics, and many do not like the subject, with some even admitting a "fear" of mathematics. If such fearful students further hold the intelligence-is-fixed view, their confidence and perhaps their willingness to expend effort are likely to be low.

Accordingly, we gathered data on the students early in the semester. Responses of students in the control class were about the same as those in the experimental class, so means for the responses of all students are reported here, with 5 and 1 representing the positive and negative poles, respectively. (The dubious practice of averaging Likert items is acknowledged, so means are offered only as an effort to summarize group performance. Scores on all the items are included in Appendix B.5, along with the complete questionnaire.)

The following questionnaire items address the two views of intelligence; items were adapted from Dweck (Dweck, 1987; Dweck & Leggett, 1988; Elliot & Dweck, 1988) and Elliot (1986). Students checked one of *Agree, Slightly agree, Undecided, Slightly disagree*, and *Disagree*. Item numbers reflect their position in the complete questionnaire (28 Likert items, plus two open-response items). The +/- following the item indicates how the item was regarded in recoding; items marked with - were recoded (via 6 - marked score) so that results on different items could be combined in a consistent fashion.

2. Intelligence is something you can increase if you want to. +
4. You can learn new things, but how intelligent you are stays pretty much the same. -
6. If a person is not good in math, it doesn't matter how hard he/she works. -
11. You can change how intelligent you are in math. +
25. Hard work can increase my ability to do math. +

Mean performance of the 53 students was 4.2, with only five students giving a mean indicating that they were undecided or leaned toward the intelligence-is-fixed view. Hence, the students in the large had what most instructors would view as the better outlook: Working hard can make a difference.

Learning goals. Another attractive hypothesis (Dweck, 1987) is that learners may have two types of goals when they are studying mathematics: performance goals, and mastery goals. Students who value performance goals most highly favor tasks that will enable them to perform well by some external standard--grades, parental or teacher approval, regard of classmates, etc. In contrast, students with mastery goals look for internal evidence that they understand the material being studied. Most students probably have both types of goals, of course, but it is relatively easy to find students at the extremes, especially at the performance end. Here are the questionnaire items directed toward the performance vs. mastery dimension, with a summary report as before. Responses were weighed so that at the extremes, 5 would reflect mastery goals, and 1, performance goals.

14. I prefer hard, new, and different tasks so I can try to learn from them. +
17. I like things I'm good at so I can feel intelligent. -
19. I prefer tasks that are fun and easy to do, so I don't have to worry about mistakes. -
22. I like problems that are hard enough to show others that I'm intelligent. -
26. I like problems that aren't too hard, so I don't get many wrong. -
28. I would rather have someone give me the solution to a hard math problem than to work it out for myself. -

Mean performance of the responding students was 2.9, with only three students having a mean suggesting at least a slight interest in mastery goals. This result is sobering since it suggests that preservice teachers are more concerned about grades in mathematics, for example, than a genuine understanding of mathematics. Hence, the primary emphasis in the experimental class on the understanding of concepts, acknowledged to be more difficult than mastery of skills was perhaps counter to the students' orientation. To be fair, some preoccupation with grades is understandable, since certain grade point standards are necessary for admission to the teacher credential program at San Diego State University.

Confidence. Items to attempt to determine the degree of confidence of the students were drawn largely from Kloosterman (1986).

1. I am sure of myself when I do math. +
9. I'm not the type to do well in mathematics. -
12. I don't think I could do well in a calculus level course. -
16. I'm not good in math. -
21. It's always better to use a calculator for calculations if one is available. -

Mean performance on these items for the 53 students was 3.2, indicating a fairly neutral position overall on confidence. This is somewhat deceptive since there were only 13 of the 53 with mean scores between 2.5 and 3.4. Twenty-three students had means 3.5 or greater, with 17 having means less than 2.5.

Other dimensions. The following dimensions were treated by fewer items on the questionnaire. Since they may not be well represented here, results for them are especially suspect. Dimensions represented by only one item (Thoughts about success, item 8, and Thoughts about failure, item 23) are not reported here.

Failure as acceptable. A more demanding curriculum will result in more frequent lack of success. An interesting dimension then is one's willingness to accept failure. Items were selected from Kloosterman (1986).

3. Good math students almost never get problems wrong. -
15. Making a mistake shows you are dumb in math. -

Mean performance on these two items was a pleasing 4.5.

Enjoyment of mathematics. Two items attempted to address this dimension.

5. I don't understand how people can enjoy spending a lot of time on math. -
13. Math is fun and exciting. +

Mean performance on these two items was 3.3, somewhat disappointing in a group of preservice teachers who will be teaching mathematics every day.

Perseverance. This dimension is recognizably important for a course that deliberately includes tasks intended to lead to cognitive dissonance (cf. Fennema & Peterson, 1983).

7. Once I start trying to work on a math puzzle, I find it hard to stop. +
27. If I can't solve a math problem right away, I stick with it until I do. +

Mean performance on these two items was also 3.3.

Mathematics as a male domain. Preservice elementary teachers are predominantly female, so an endorsement of this dimension would be quite serious, both for the preservice teachers and for their future students.

10. Males are by nature better at math than females are. + (meaning agreement with the undesirable "Mathematics as a male domain")
20. Mathematics is as important for females as for males. -

Fortunately, the mean for these two items was 1.3, indicating disagreement with a view that certainly could be harmful to a female student or a teacher of either sex. Fifty-one of the 53 students had means indicating either disagreement or slight disagreement.

Mathematics as mechanical. Two items attempted to determine how the students viewed mathematics.

18. Mathematics problems always have just one right answer. + (meaning agreement with the "mathematics as mechanical" position)
24. There is always a best way to do things in mathematics. +

The mean for these two items was 2.4, somewhere between "Undecided" and "Slightly disagree." One would hope for low scores on these two items. Perhaps the 10 students with means above 3 (meaning agreement with the statements) were thinking of the many algorithmic tasks in their past mathematics experience and not the broader and richer type of mathematics task we, and the 34 students with means less than 3, might have in mind.

Summary. The students seemed to have views that would encourage them to work hard, although driven more by a concern for grades than for learning. Their confidence levels were quite varied. One might hope that their somewhat neutral attitude toward the enjoyment of mathematics and toward perseverance would be more positive. Roughly two-thirds of them recognized that mathematics is not necessarily rigid. Fortunately, these 53 students did recognize that failure can be expected on mathematics tasks and, in particular, did not view mathematics as a male domain.

Results from Final Questionnaire on learner attitudes

The questions and a summary of responses for each question are included here. The entire set of responses is included in Appendix B.6.

1. What one or two topics do you feel you made most progress on, and why?

Most of the students felt they had profited most from the work on fractions. They felt they had a better understanding of fractions, could use benchmarks when comparing and estimating with fractions, and felt more confident of their ability to operate with fractions. Two students said they were no longer anxious or uncomfortable with fractions, that one could use "common sense" instead of just rules when working with fractions. In this same vein, two said they could now make sense of percentages and estimate percents. About a third of the students also said they had made progress on probability, or probability and statistics, because these were new topics for them.

2. What topics or concepts do you still feel uncomfortable with? Can you explain why?

About a third of the students remained confused about probability concepts, and an overlapping third were still unsure of some of the statistics concepts. One mentioned that the "correct probability doesn't always look correct" and that the wording is important. Two felt we had "pushed" through these topics. Others mentioned fraction multiplication or division (e.g., the difference between 5 x 1/4 and 1/4 x 5), ordering numbers (fractions), and word problems (which focused on understanding operations).

3. Many people believe that doing mathematics is basically learning and following a set of rules. Do you agree or disagree?

Students overwhelmingly disagreed with this statement, but many felt that they had only come to this understanding in this class. Statements such as "At the beginning of the class I would have agreed with this but now I know that math is much more common sense"; " If one can use number sense and estimation abilities, one can do mathematics"; " I disagree, but it is still hard for me to get away from following the rules"; "As a senior in college, I'm finally learning the reasons behind the rules"; "In this class I noticed that mathematics involves ideas, primarily understanding of what you are doing" permeate the students' responses to this question. Only one student admitted that she still basically agreed with this statement: "I still do not feel I am good with math, but what I know well are formulas."

4. How did group work help or hinder you in learning mathematics in this course?
5. Do you feel that you participated fully in group work? Did others in your group participate fully?

All students but one found group work to be helpful, primarily because they were able to see many different approaches and solutions to problems, and because they felt comfortable asking questions and taking risks with peers. The lone dissenter felt that she never knew what was right or wrong when there were many ideas floating around, and that some people took control and left others out.

All felt that they had participated in group work, but two held back when they did not fully understand the problems. A few said that others had not participated as fully as they could have.

6. Did you make any special efforts to try to make sense of the mathematics in the take-home quizzes? If so, how?

Most students responded positively to this question. They felt that the take-home quizzes allowed them the time to work through problems carefully, using drawings and even cut-outs to help in understanding, and that they had profited more from take-home quizzes than from in-class quizzes.

(We deliberately gave take-home quiz problems that called for the type of exploration that required more time than usually allowed in quizzes given in a class setting. The problems could not usually be answered simply by reviewing notes or the text.)

7. How frequently did you ask yourself if your understanding was adequate for a teacher of mathematics?

Most students said that they often considered this question, particularly when a concept was new or had been confusing to them.

Knowledge Measures

Student learning was measured through written questionnaires, through interviews, and by performance on the final examination for the course. The written questionnaire (RNQ) was a 47-item test of understanding of rational number and proportional reasoning concepts. Students completed the questionnaire during the first week of the course and again after the units on these topics had been completed. This questionnaire was also given before and after instruction to a control class that was taught by another instructor, using the traditional text for the course. The questionnaire together with performance data can be found in Appendix B.7.

Each student in the experimental class was interviewed during the first and last weeks of the course. The interview consisted of questions measuring rational number understanding and some standard probability and statistics concepts. Except for a few items deleted from the first interview because they were not informative and one item added to the final interview, the questions were the same for the two interviews. Both interview forms appear in Appendix B.8.

The final examination was comprehensive and can be located in Appendix B.4. The measurement and LOGO items are not discussed here.

Several of the items measuring rational number and proportional reasoning concepts were adapted from a survey of middle school mathematics teachers by Post, Harel, Behr, and Lesh (1988). Some of the probability and statistics items have been widely used in research studies and are discussed by Shaugnessy (in press).

Rational number concepts.

As noted earlier, much of the instruction in the rational number unit focused on rational number meaning. Ability to make different interpretations of rational numbers, ability to represent, order, and compare rational numbers, and the ability to mentally compute and to estimate with rational numbers are all measures of understanding.

Representing rational numbers. Several questions explored the part-whole notion of fractions. In the rational number test, students were asked to tell how much of a somewhat irregularly shaded rectangle was shaded and to identify 2/3 on a number line. Percent correct on the first item increased from 73% to 100%, and on the second item from 82% to 90%. On another item, students were given 6 squares, told that they represented 3/2 of some whole amount, and asked how many squares were in the whole. The number of students who broke the 6 squares into three subsets of two, then took two of them to give the answer 4 increased from 41% to 76% in the experimental class, and from 44 to 57% in the control class. A small number in each class solved the problem by setting up a proportion.

A similar question was asked during the interviews. On the initial interview, students were given 9 x's and told that they represented 3/4 of the whole. All but one student could identify the whole as 12. In the final interview the same information was given, but the problem was more complex in that students were asked what 2/3 of the whole would be. Again, all but one student could answer correctly. The primary method was to divide the 9 x's into three subsets, add another subset of three, then divide the entire 12 x's into three parts and count the x's in two of those three parts.

The division interpretation of a fraction was explored in the rational number test through an item that showed three circles representing pizzas; students were asked how the pizzas could be shared by eight people. The most common response, in both experimental and control classes, and both before and after instruction, was to draw lines dividing each circle into eighths, with each person taking one piece from each, yielding 3/8 of a pizza. The only real difference (from 5% to 19%) from pretest to posttest was in the number of students who cut two of the pizzas into 4 pieces and the third into eighths, but again gave 3/8 as the answer.

Another aspect of this factor is the ability to ignore distracting information. For example, students were asked on the RNQ to show how a candy bar, marked in thirds, could be shared by four people. The number of students in the experimental group who could ignore the "third" marks increased from 36% to 62%. An additional 34% used a correct procedure of dividing each of the thirds into fourths. Only 54% of the experimental group were able to do this problem correctly on the posttest. On a more difficult item, students were required to shade in 3/8 of a circle on which thirds had been marked. Students able to ignore the thirds increased from 27% to 43% in the experimental class, but decreased from 19% to 4% in the control class. In addition, some students in both groups broke each third into eighths and shaded in 3/8 of each third.

The ability to ignore distracting information was also examined during the interviews. In one item, students were asked to shade in 2/3 of a rectangle that was divided into fourths with a horizontal and a vertical line. Only 8 of 22 students in the initial interview and 12 of 21 in the final interview were able to ignore the distracting lines. Some used the extra lines to their advantage, adding four vertical lines to get six parts (ignoring the horizontal line) and shading in two, or adding lines to obtain 12 parts and shading in eight. Some students, however, could not do this problem: "I can't get 2/3 from four (final interview);" "It's between 1/2 and 3/4 (proceeds to shade in two of the small rectangles and part of a third);" "It has to be cut in thirds (proceeds to draw a new rectangle without the distractors, mark thirds, and shade in two of them)." In a second item, students were shown a diagram of 1/3 of a circle, told in the initial interview that it represented a half of a whole, and asked to draw the whole. All but one could do this correctly. On the final interview, students were told that the diagram represented 3/4 of a whole and were asked to draw the whole. The large majority of students divided the given section into three equally-sized parts, then added a fourth part of the same size.

The decimal representation of rational numbers is dependent upon the understanding of place value. This understanding was tested by two items on the RNQ. Students were asked to correctly place the decimal point in 4.5 x 51.26 = 023067000. The percentage of students in the experimental class who counted three decimal places decreased from 64% to 43%, but a few of those who were correct felt they had to first preform the complete multiplication. A similar problem was given during the interviews, with similar results. On another item in the RNQ, students were to find 1000 + 10.001 + 0.01. The number of students who needed to write out the numbers in vertical fashion and use the addition algorithm decreased from 82% to 48% in the experimental class, and from 81% to 57% in the experimental class.

Summary. By the end of the term, students in the experimental class showed a good understanding of the part-whole notion of fraction, but could become confused with drawings where distracting lines were present. Most still did not think of a fraction as a representation for division. The majority were able to use place-value understanding to solve problems with decimal numbers.

Ordering and comparing rational numbers. During the pre- and post-interviews, students were asked to order the numbers 7/8, 0.31, 1/3, 0.2, 0.75, and 1/4. The number of students able to order these numbers correctly increased from 12 to 17. During the final interview three students incorrectly ordered 1/4 and 0.31 or 1/3 and 0.31, while one student (who failed the course) remained very confused about ordering rational numbers. During the initial interview four students ordered the fractions and the decimals separately, but no one did so during the final interview.

A final exam question required students to order 1 3/5, 95%, 15/12, 145%, 1 14/21, 1 14/22, 1.82%, and 1/17. Only eight students could do so correctly; another eight had one number out of place, while five had two or more out of place. Errors usually occurred when ordering 1 14/21 and 1 14/22, or in placing 1.82% correctly.

Six items on the RNQ asked students to identify the largest of three or four numbers. On the item with four numbers, 1/9, 2/3, 3/11, and 14/28, the percentage of students in the experimental class who had the item correct moved from 77% on the pretest to 95% on the posttest, while the control group moved down from 78% to 75%. Two other items on the RNQ asked students to tell what happens to a fraction when the numerator increases and the denominator decreases (easy for all students) and when numerator and denominator are both increased (very difficult for students even on the posttest).

Summary. By the end of the term, students could use benchmarks to assist them in ordering rational numbers and had a much better understanding of the sizes of the numbers. However, they continued to be confused when comparing very small percents to other numbers, fractions and decimal numbers that are close to one another, and numbers that vary only in the denominator, such as 14/21 and 14/22.

Density of rational numbers. On the RNQ, students were asked to find a decimal number, if possible, and then a fraction, between 0.7 and 0.8. On both the pretest and posttest, students were quite able to find an appropriate decimal. Several then changed the decimal to a fraction. 91% of the experimental class and 74% of the control class were successful. Finding a fraction and then a decimal between 1/4 and 1/5 was slightly more difficult, but the number correct increased substantially in both classes.

Using number sense to compute. Many times, simple computations can be done quickly using knowledge of numbers instead of the computational algorithms. Eighteen items on the RNQ tested this ability. For example, 5/8 + 9/18 could be done quickly by recognizing the 9/18 is 1/2, or 4/8. On this problem the number of students who appeared to use number sense to solve the problem increased from 23% to 52% in the experimental group, but stayed at about 10% in the control group. The number of students who elected to solve the problem by finding the least common denominator of 8 and 18 decreased from 59% to 19% in the experimental class, but only from 66% to 61% in the control class. In most cases, students in the experimental class appeared to used some number sense to solve posttest items when possible.

During the interviews, students were asked: 12 is 3/4 of ? The number of students who thought of this problem in terms of breaking 12 into 3 groups of 4 and then adding another group of 4 increased from 4 to 14, while the number of students who attempted to set up an equation or proportion to solve the problem decreased from 7 to 5. Students were also asked to find 1/2 + 0.5. The number of students who recognized this to be 1 increased from 8 to 14, while the number of those who tried to solve the problem algorithmically decreased from 12 to 5. Students were also asked to do two problems mentally during the interviews. For 85 - 0.3, a disappointing 13 on the initial interview and 12 on the final interview attempted to carry out the mental analogue of the paper-and-pencil algorithm: "Line up the decimal..."; "I'd say eighty-five point zero, bring this over...." For 0.6 + 2.101 + 4.42, ten students on each interview used the standard algorithm or a slightly changed version of it.

Summary. Students demonstrated more number sense at the end of the term, but continued to use the standard algorithm on mental computation problems.

Estimation. The RNQ contained two computational estimation items, each with a multiple choice response. The number of students choosing 1 as the estimate of 0.47 x 2.1743 remained at about 75%, with the most common distractor being 0.981921. The number selecting 2 as the best estimate of 12/13 + 7/8 increased from 64% to 86% for the experimental group.

Eight estimation items were included in the interviews. Only those with interesting differences between initial and final responses are discussed here. For 5.8 ÷ 12, the number of students who estimated this as 6 ÷ 12 increased from 6 to 10, and the number who estimated it as 4.8 ÷ 12 increased from 2 to 5. In estimating 5/6 + 7/8 + 1/15, all students on the final interview estimated the answer as 2 or close to 2, while half of the students said they could not do it for the initial interview. The number of students who estimated 0.57 x 789 as 1/2 x 800 increased from 9 to 15. The number of students who recognized that 1 is a good estimate for 2/3 x 1.07 increased from 2 to 8, while the number who said "sixty-six hundredths" was the answer because that is the same as 2/3 increased from 3 to 8.

The final examination contained two estimation problems. Almost all students were able to estimate 97% of 17.85. (Two incorrect responses had the decimal point in the wrong place.) Fifteen students were able to estimate 5.1 ÷ 0.33 by saying that there are 15 thirds in 5.

Summary. Students' ability to use benchmarks influenced the manner in which they undertook some computational estimation problems. Where it was not possible or appropriate to use benchmarks, there were few differences between performances at the beginning and end of the term.

Meanings of the operations.

It is well known that, although most preservice elementary teachers can carry out computations, their meanings for the operations are limited. Consequently, they may have difficulty in dealing with such applications of the operations as are represented by the usual story problems and in representing the operations with manipulative materials common to the classroom. Both deficiencies are serious, since most recommendations call for increased use of both manipulative materials and applications of mathematics.

Modeling the operations with materials. The final examination (see Appendix B.4) included some items to determine whether the students could interpret the operations for given materials (the students made drawings of the work, without having the materials present). Results were mixed. Least encouraging was the performance in showing 1.2 – 1.03, using place-value materials and the missing-addend view of subtraction. Although roughly half of the students were correct, one-third made drawings involving a take-away view, and 10% made drawings involving the comparison view (the other 5% missed the item completely). Student performance on other items was promising. The students did quite well, as would be expected, at making a drawing to show $3 \times \frac{3}{4}$ (90% correct). Their performance on showing how many $\frac{2}{3}$s are in $2\frac{1}{3}$ was also good, with 95% correctly writing an equation for the work. Their performance on two transfer items dealing with number line representations was also encouraging, both in showing $\frac{1}{3} \times \frac{3}{4}$ on a number line with 0 to 4 shown (81% correct), and in showing the more difficult $2\frac{1}{4} \div \frac{1}{2}$ and interpreting the work (71% correct).

Summary. The students' performance suggests that we should continue to incorporate work with manipulatives into each mathematics course for preservice teachers. Such work "forces" them to attach meanings to both the numerals and the operation signs, in contrast to carrying out computational algorithms already known.

Solving application problems (story problems). The pre- and post-questionnaire (RNQ) (Appendix B.7) included six story problems for the students to solve, with some included to identify common misconceptions. Performance on a comparison subtraction item (#25) grew from 60% on the pretest to 91% on the posttest, even though the control group's performance improved only slightly (from 71% to 75%).

An item (#20) leading to $40 \times \frac{5}{8}$ was quite easy (95% pretest, 100% posttest), although the students showed only a little improvement in giving the canonical order (from 41% to 48%), something that was given attention during instruction; apparently computational conventions rather than operation meaning continued to hold sway. It is interesting (and dismaying) that the control group's performance declined on this item (from 95% to 81%)! Performance on a comparison multiplication problem (#23, involving $\frac{3}{8} \times 3$) improved considerably, from 41% to 81% (with the controls improving from 47% to 59%). Roughly a quarter of each group chose to divide on the pretest, presumably falling victim to the well-known but limited "division makes smaller" misconception; they expected the answer to be less than 3 and thought, instrumentally, that division was the proper means to get such an answer. Fortunately, only 5% of the experimental group (and 11% of the control group) chose division on the posttest.

The multiplication mal-conception, "Multiplication always makes bigger," was tested by item 22, involving 0.78 pound of cheese at $2.46 a pound; performance improved from 64% to

82% (control: 48% to 62%). Yet 14% persisted on the posttest in choosing division; the control group's figures went from 41% choosing division in the pretest to 30% in the posttest.

There were two problems routinely solved by division. On a repeated-subtraction division (#21, involving the number of $2\frac{1}{2}$ s in 9), the experimental group's performance declined, from 82% to 71% (control group figures: from 62% to 71%)! Item 24 (Jim bought $4\frac{1}{2}$; Sally, $2\frac{1}{2}$, Jim bought how many times as much as Sally?) showed a slight improvement, from 46% to 57% (controls: from 49% to 64%). One explanation might be that the incorrect choices of subtraction rose from 18% to 33%, perhaps reflecting the similarity of a multiplicative comparison setting (not covered in class) to that of subtractive comparison (extensively covered in class).

Summary. In the large, students improved in their ability to solve routine story problems. The novelty of the item for which performance declined suggests that a more comprehensive treatment of applications of the operations be given in the courses for preservice teachers; giving only the "picture" for grades K-6 may mislead the students into thinking that they have the complete story on story problems.

Writing story problems. One item on the RNQ asked for a story problem leading to $\frac{1}{2} \times \frac{3}{4}$. Since this had been a difficult topic for the students during instruction, it was somewhat pleasing to see an improvement from 5% to 38% or, including as correct problems for $\frac{3}{4} \times \frac{1}{2}$, from 14% to 52% (corresponding percents for the controls: 10% to 9%, or 20% to 22%). On the final exam, 81% were able to write a story problem illustrating $3 \div 8 = \frac{3}{8}$. And 71% were able to complete a story problem that was started, with the instruction to lead to $6\frac{1}{2} - 4\frac{3}{4}$ from the comparison view. On a similar item, 81% were able to complete a story problem that was to lead to $4.5 \div 0.6$ (repeated subtraction).

Summary. The students' ability to profit from work on writing story problems was encouraging. With more experience, spread over all the courses taken by the students, performance should be noticeably improved. Some students seemed adept only at imitating sample model problems for a particular view of an operation, so items which require them to respond to novel situations should be used frequently.

Proportional reasoning.

Proportional reasoning is one of the hallmarks of the Piagetian formal operations stage. The determination of a student's ability to reason proportionally is often clouded by the student's algorithmic proficiency. Some items on the pre- and post-rational number quiz (RNQ in Appendix B.7) (experimental and control), during the interviews (in Appendix B.8), and on the final examination (in Appendix B.4) addressed different facets of proportional reasoning. Instruction on proportional reasoning, in hindsight, was weak for the experimental group. Lessons featured hands-on work rather than an emphasis on an algorithm. The control group apparently spent considerably more time on writing and solving proportion equations.

Algorithmic items. In four items on the RNQ students were to find the missing term in a given proportion (4/6 = 6/x; 3/8 = x/12; 15/9 = x/12; 8/15 = x/5, using an x here for a blank on the quiz). Although the experimental group showed some improvement on these algorithmic tasks from pretest to posttest (gains of 8% to 18%), the control group's improvement was noticeably greater (gains of 17% to 29%). Hence, the control group's greater time on an algorithm for solving proportions paid off on these tasks.

Proportional reasoning items. This computational facility, however, did not result in superior performance on less conventional items. For example, the RNQ included a series of items in which students were to choose the orange juice concentrate and water mixture with more orange flavor, given information about the relative amounts of concentrate and water for two mixtures (cf. Noelting, 1980a, 1980b). The experimental group outperformed the control group by at least 13% on six of the seven items. These figures may be somewhat spurious, however, since the percent omitting the items on the posttest suggests that some students did not have enough time to finish the posttest. In any case the control group's superiority on algorithmic tasks did not appear to carry over to a non-algorithmic task. Similar remarks hold for a non-conventional proportion task (item 27 on the RNQ) and a students-professors problem (item 28).

Two other items were also intended to ferret out evidence of proportional thinking. One item, during the interviews of the experimental students, asked for a qualitative judgement about relative speeds under two circumstances: yesterday's and today's, when "Nick drove fewer miles in more time than he did yesterday." There was some growth, from 52% correct to 76%, but even on the posttest 19% thought that more information was needed (e.g., "You need values") In one item on the final examination (item 17, experimental only), the students were to identify the box giving a greater chance of drawing a black ball (box 1: 3 black, 2 white; box 2: 4 black, 3 white). Two-thirds of the students made the correct choice, with 43% showing the canonical probability approach by comparing 3/5 and 4/7 with the other 24% comparing 3/2 and 4/3. It is more disconcerting that 18% used an additive approach and concluded that the chances were equal for the two situations.

Summary. Although spending more time on algorithmic approaches to conventional proportion exercises does result in improvement on such tasks, unfortunately that improvement does not carry over to non-conventional proportion tasks. Some students apparently need considerably more experience with proportional situations than the courses (and their previous backgrounds) offered. Indeed, work with inservice teachers (e.g., Post et al., 1988) and other adults suggests that many people never learn to deal with proportional reasoning.

Statistics

The three major topics that were focused upon during instruction were measures of central tendency, measures of deviation, and use of different graphing techniques.

Central Tendency. On the true-false item stating that the overall GPA of a student is 3, given that the student had a GPA at one school of 3.2 and a GPA at another school of 2.8, only 8 of 21 students recognized that the information was not complete and marked the answer as false. A final interview item asked for possible prices on eight bags of potato chips of the same weight, given that $1.39 was the average price. Only eight students exhibited a clear understanding of what the prices could be. When asked if it was possible to have 7 bags priced below (or above) $1.39, students generally hesitated, said no, then changed their answer to yes, given an extreme price in the other direction. However, they did not recognize that not any extreme price was possible. Two thought that there had to be four bags priced above $1.39 and four bags below.

An interview item required that students reflect on the difference between the mean and the median. Students were told that the real estate section of the newspaper usually gives the median house price rather than the mean house price. They were asked why they think this is done. The most common answer was that the median is less, and therefore more attractive. A few students recognized the appropriateness of the median: "Because some houses in La Jolla would be outrageous, and that could set off the average. ("What price would you give to a relative coming here to house hunt?) I wouldn't want those houses considered...I'd tell them the median." Most students, however, appeared quite confused about the information portrayed by these two measures. Even when they recognized that the median is not going to be affected by outliers in the same way as the mean is, some continued to feel that the median was in some way a "dishonest" price: "The mean is influenced by outliers, but not the median. The median looks better. More people look like they can afford it. (Which price would you give to a relative coming here to house hunt?) The mean. It's more realistic."; "Because the average (mean) would be too high. It would scare customers." and "It would be deceptive I think." Answers also portrayed other misunderstandings about the median: "(What if I said the median was $208,000?) That would mean the prices go from $0 to $416,000."

Deviation. Understanding of deviation was examined through true-false items on the final exam. Although students showed a basic understanding of quartiles, they associated percentiles with percents: 15 of 21 said that a student who answers 3 of 5 questions correctly would score at the 60th percentile. Even after instruction, z-scores remained a mystery to most students.

Graphing. Even though they were given many opportunities to write interpretations for graphs, many students found this to be an extremely difficult task. On the final exam, they were given a box-and-whiskers graph of ages of teachers in two school districts and asked to write an interpretation. Although students could readily identify significant information from the graph such as the median, the quartiles, and the range, about half could not relate this back to ages to give some meaning to the numbers. On another problem, students were given information about grams of fat in fast foods and asked to make a stem-and-leaf graph of the data and write an interpretation. Again, although the graphs were correct for the most part, the interpretations were poorly written.

Summary. Although students readily learned how to find measures of central tendency, average deviation, and z-scores, their understanding of these concepts was superficial at the end of the instructional period. Students appear to need more time, practice, and in particular, work on problems which force superficial understandings and misconceptions to surface. Their difficulty in writing interpretations for graphs points to a more general difficulty with communicating mathematically.

Probability.

The formal study of probability is new to most students in the courses for preservice elementary teachers. They have, of course, the usual common knowledge of, and exposure to, situations and everyday language that is probabilistic in nature, along with such misconceptions as the gambler's fallacy (e.g., one is more likely to win after several losses). It is interesting, then, to see what learning occurs with formal instruction and how well the instruction combats such misconceptions. We have data only on the experimental group, some from the final examination (in Appendix B.4) and some from the interviews (in Appendix B.8).

The learning of techniques. It was encouraging to see that the students did learn, at least at the time of the final examination, how to handle fairly complex probability problems. Item 21 on the final examination involved a binomial experiment; 78% of the students received full credit and 14% received no credit for the problem. For another multistep experiment (item 25) 81% could make an appropriate tree diagram (the method used in the course), with 88% of those students successfully finding the probability of an event for the experiment. Hence, as is usual, the students did attain fairly well on many algorithmic aspects.

But even this learning was not complete. For example, only 19% were successful on an item (23) involving an application of the fundamental counting principle (possible colorings of a 5-region drawing with 6 colors, allowing repetitions)! To be fair, this type of problem is often difficult for students, with many trying to memorize a rule about raising one number to a power (24% gave the answer 5^6) rather than using an analysis closer to the fundamental counting principle (28% gave 6x5, an answer of a form commonly encountered in such problems, and so an unthinking use of the fundamental counting principle). Similarly, the students' performance on a non-conventional tree diagram problem (item 27) was mixed (48% correct) even though most could deal with the tree diagrams in the conventional problems mentioned above. And, once again *when* to use an algorithm seems to have taken second billing to *how* to carry out the algorithm: 38% thought that Pascal's triangle could be used in any multi-step experiment.

Conceptual knowledge of probability. There were some encouraging signs. For example, 81% recognized that tossing a fair coin 80 times could result in 36 heads. And, 90% knew that after 5 consecutive heads on tosses of a fair coin, the probability of heads on the next toss was 1/2.

But conceptual grasp in other areas tested was less clear. In a transfer item (22), when asked whether an insurance company's determination of the probability of a car accident for someone of a given age and sex was an experimental probability or a theoretical probability, 28% thought experimental and 56 thought theoretical. There seemed to be little growth in an interview item (24) asking, for example, what a weather forecast of "70% chance of rain" meant. And on the final examination 48% thought that each outcome in a 4-outcome experiment is always 1/4; a rather dismaying result since the students had repeated exposures to experiments with unequally likely outcomes.

In some cases, students may have learned to give "correct" answers without really believing them, a most tenuous type of learning and one undesirable in efforts for a sense-making curriculum. For example, one interview item (22) asked the students to discuss the relative likelihood of three ordered five-card hands (one involving 4 aces, another 3-7 in hearts, and the third a mix of suits and values). There was a substantial improvement in the recognition that each hand was equally likely (from 18% on the first interview to 52% on the second), but one student offered, "That's just theory; that's not real!" Or, in a similar item dealing with two orders of sexes in 6-children families, one student gave the correct answer and implied that the answer was contrary to "instinct." In response to the interviewer's "So you don't go by instinct any more?", the student replied, "Not since this class." Whether this new mistrust of intuition was healthy, or whether it meant that the student merely gave the *opposite* answer to the intuitive one, is not clear.

Sampling. One area for which we did not have enough time was that of sampling. One item (26) in the first interview showed the students' deficient understanding. The setting was a mix of 100 engineers and lawyers, 30% engineers, 70% lawyers. A person was drawn randomly and described in terms opposite to those stereotypic for lawyers. Only 14% of the students stated that a lawyer was more likely for the person drawn; others ignored the numbers and focused on the characteristics ("Engineer--I don't see a lawyer as conservative." "He's an engineer; he's not interested in political issues.").

Summary. Perhaps reflecting the students' view that doing things is more important than understanding them, their performances on algorithmic items was acceptable, particularly since these techniques were new to the students. But the more difficult conceptual grasp, particularly a firmly entrenched one, was not so widely reached and may have been tentative for many students. It is likely that a "first shot" at understanding probability ideas needs a follow-up, perhaps in a subsequent course, or if the luxury of a later course is not available, a more extensive or frequently-revisited treatment in a single course. Limited data, and expositions such as that of Paulos (1988) suggest that ideas of sampling require attention.

Discussion

Instructor impressions of students

All but one student was employed, and it was our feeling that heavy work schedules were detrimental to learning. The mean number of hours worked per week was 19. The long working hours together with heavy class schedules limited the amount of time students devoted to this course outside of class. Students did not devote sufficient time to assignments, reflections, and preparation for group work. According to a survey made at the end of the term, many students spent only about two hours a week on homework and take-home quizzes. Even though there was considerable progress made by many of the students, the instructors felt that for the most part many of the students remained unwilling or perhaps unable, given the time constraints, to explore problems beyond a superficial level, and that their knowledge and skills at the conclusion of the course were less than satisfactory. Six students received a B as a final grade, ten received a C, four received a D, and one student failed the course. These are lower grades than either instructor has given previously in content courses for elementary teachers.

References

Dweck, C. S. (1987, April). Children's theories of intelligence: Implications for motivation and learning. Paper presented at the Annual Meeting of the American Educational Research Association, Washington.

Dweck, C. S., & Leggett, E. L. (1988). A social cognitive approach to motivation and personality. *Psychological Review, 95*(2), 256-273.

Elliot, E. S., & Dweck, C. S. (1988). Goals: an approach to motivation and achievement. *Journal of Personality and Social Psychology, 54*(1), 5-12.

Fennema, E. & Peterson, P. (1983, April). Autonomous learning behavior: A possible explanation. Paper presented at the annual meeting of the American Educational Research Association, Montreal, Canada.

Kloosterman, P. (1986). Attitudinal predictors of achievement in seventh-grade mathematics. In G. Lappan & R. Even (Eds.), *Proceedings of the Eighth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 244-249). East Lansing, MI: Michigan State University.

Noelting, G. (1980a). The development of proportional reasoning and the ratio concept: Part I - Differentiation of stages. *Educational Studies in Mathematics*, *11*, 217-253.

Noelting, G. (1980b). The development of proportional reasoning and the ratio concept: Part II - Problem structure at successive stages; problem solving strategies and the mechanism of adaptive restructuring. *Educational Studies in Mathematics*, *11*, 331-363.

Paulos, J. A. (1988). *Innumeracy: Mathematical illiteracy and its consequences*. New York: Hill and Wang.

Post, T. R., Harel, G., Behr, M. J., & Lesh, R. (1988). Intermediate teachers' knowledge of rational numbers concepts. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 194-217). Madison, WI: Wisconsin Center for Education Research.

Shaughnessy, J. M. (in press). Research in probability and statistics: Reflections and directions. To appear in D. Grouws (Ed.), *Handbook for Research in Mathematics Education*. New York: Macmillan.

PROCESS AND INQUIRY IN LIFE SCIENCE

by

Kathleen M. Fisher, Cheryl Mason, and Phoebe Roeder

Overview

Background

Process and Inquiry in Natural Science (NS412) is a hands-on science course for Liberal Studies majors (prospective elementary and middle school teachers). From its creation more than ten years ago, NS412 has incorporated many principles of cognitive and constructivist psychology. The course provides two integrated lecture-laboratory experiences per week that are designed to a) involve students in doing science, b) model for students ways in which they can effectively teach science in their classes, c) help students overcome their fears of and/or dislike for science, d) engage students in meaningful in-depth learning of key scientific concepts, and e) help students acquire basic scientific skills. Three versions of the course are offered, in life science, physical science, and earth science and astronomy. The study described here focuses on the life science version (NS412C).

Life science versions of the NS412 course are based in part on activities from the Science Curriculum Improvement Study II (Abruscato, Fossaceca, Hassard, & Peck, 1980) and the Elementary Science Study (Alberti, Davitt, Ferguson, & Repass, 1976). The pilot study described here restructures a life science section so as to make the content more suitable for middle school teachers and to employ more strategies for promoting conceptual understanding.

Course Planning

During fall 1989, the authors discussed general instructional issues and strategies in the CRMSE preparatory seminar, designed the biology curriculum for the experimental section, and discussed domain-specific instructional strategies. The following course goals were established.

Cognitive Strategies

A primary aim was to stimulate and facilitate the construction of rich mental models by students, with many linkages between ideas -- to focus on depth rather than breadth. In order to engage students in constructing such mental models, specific strategies would be adapted from Scardemalia (Scardemalia, Bereiter, McLean, Swallow, & Woodruff, 1989) and Anderson (1983), as summarized in Table 1.

Table 1
Cognitive Strategies

1. Relate topics to personal knowledge and experience.
2. Examine discrepant events and consider alternative explanations.
3. Minimize working memory load by using external memory aids.
4. Distinguish between observations and inferences.
5. Use estimation and checking strategies to assess ideas.
6. Practice new skills, improving through successive approximations.
7. Actively participate in class and contribute to one another's learning.
8. Make knowledge-construction activities overt (talk about them).
9. Review often, creating more elaboration and links to other topics.
10. Organize knowledge several different ways.
11. Treat knowledge lacks in a positive way and obtain feedback often on knowledge, skills, and cognitive strategies.

Content Competencies

We felt that biology at the elementary and middle school levels should focus on developing a rich understanding of key *protoconcepts* that can facilitate student understanding in subsequent biology courses. The particulate nature of matter is, in our view, an important protoconcept. Thus, some emphasis was placed on such abstract ideas such as 'molecule' and 'diffusion' Additional content competencies are summarized in Table 2. The unifying content themes of the course were the characteristics of living things (Stepans, 1985) and the particulate nature of life processes. The course also focused on developing the metacognitive skills required for organizing biology knowledge and for comprehending a semantically complex domain.

Table 2
Content Competencies

Briefly describe -

1. Key interactions between organisms and their environments.
2. Organismal development and life cycle.
3. The role of photosynthesizers in capturing energy
4. The flow of energy through a community.

5. The cellular basis of life.
6. The chemical basis of living things, especially the nature and functions of four molecules (oxygen, carbon dioxide, water, and glucose) and the general structure of four major classes of molecules (carbohydrates, lipids, proteins, and nucleic acids).
7. Human organ systems, especially the respiratory, circulatory, urinary, and digestive systems.
8. Mitosis and meiosis.
9. Mendelian (single gene) and quantitative (multigenic) variation.
10. Evolution.

Affective Attitudes

Our experience with preservice elementary students is that they tend to fear science and consequently delay taking NS412 as long as possible. Because many elementary and middle school teachers lack confidence in their understanding of the nature of science, they tend to teach it (if at all) as facts to be memorized, using textbook, worksheets, and lecture. It is hoped that by encouraging these future teachers to be confident in their ability to do science and to strive to be life learners, they will be better science teachers. The affective attitudes we aim to foster are enjoyment of biology learning, valuing meaningful understanding over rote learning, and acquiring the confidence to teach biology.

Science Process Skills

Finally, as the earlier rationale indicated, this course aims to transmit a variety of science process skills to students including ability to: 1) actively explore and ask questions, seeking alternative points of view and multiple sources of evidence, 2) observe, gather, and organize information, 3) make inferences and discriminate between observations and inferences, 4) describe in a general way and in a few specific instances how we know what we know in biology, 5) connect everyday and laboratory observations with biological theory, 6) use the metric system of measurement, and 7) use current events (e.g., newspapers articles, TV programs) for ongoing biology learning.

Materials & Methods

The experimental section of NS412C was taught by Fisher and Mason. Performance of students in this section is compared with that of students in two 'comparison' sections taught by Roeder. The comparison sections but not the experimental section emphasized the following process skills: 1) distinguish between controlled and responding variables in experiments and design controls for experiments, 2) construct and interpret graphs, and 3) develop exhaustive and hierarchical classification systems. Students in all three sections received a common content and process skills pretest (Appendix C.2) and six common questions on their final examination (Appendix C.5). They also completed a demographic survey, four presurveys (Appendix C.1) and one postsurvey (Appendix C.4). Students in the experimental section also completed weekly journals. All three sections employed collaborative hands-on activities with occasional minilectures. Students in all three sections usually worked together in groups of four. The specific course content and the emphases placed on process skills were different in the experimental and comparison sections. Some sample lessons from the experimental class are included in Appendix C.3.

Results

Biology Subject Matter
(Experimental and Comparison Groups)

Students initially had relatively poor knowledge of biology, but posttests indicated that students' biology knowledge increased significantly in all sections. This can be seen, for example, by looking at student performance on the six questions that were asked on both pretest and posttest in the experimental and comparison sections (Table 3).

The six questions were multiple choice with multiple correct responses. They covered topics taught to some extent in both the experimental and comparison sections. Students were asked to mark all answers that correctly completed an initial statement. For example, the question on biological molecules was: "Nitrogen occurs in which class(es) of molecules? a) carbohydrates b) proteins c) nucleic acids d) lipids." Both proteins and nucleic acids were considered correct because all subunits of these molecules contain nitrogen. (Some lipids such as phosphatidyl ethanolamine also contain nitrogen, but in general lipids do not contain nitrogen). The percentage of the students giving a correct response to each answer a-d was tallied. Then the average percentage of correct responses for the question was calculated for each section.

Table 3
Pre- and Post-Performance on Biology Content Questions (Experimental & Comparison Sections)

Average Percentage of Correct Responses

	Biological Molecules	Cells	Arteries	Respiration	Food Webs	Decomposers
Experimental Section	53/ 81*	38/ 75	31/ 92	75/ 91	49/ 72	55/ 68
Comparison Section 1	65#/ 72	34/ 64	26/ 79	85/ 96	34/ 78	50/ 73
Comparison Section 2	61#/ 74	27/ 62	17/ 69	76/ 88	38/ 81	53/ 71

*The top number refers to the pretest. The bottom, to the posttest.

#Possibly high because an activity on biological molecules had been performed prior to administration of the pretest.

Two technical problems might have affected the above results: First, as noted above, some instruction was presented before the pretest was given. Second, it is possible that on the pretest as many as half of the students didn't understand that they should mark all the correct answers. To assess this possibility, pretests on which more that one response was given on at least one question were analyzed; the results suggest that performance on one question ("cells") may be artificially low. Overall, these two technical difficulties do not affect the conclusion that the background knowledge of the students was fairly sketchy and that significant gains were made.

As expected, students did best on subjects that were emphasized in their section. Students in the experimental section did better on the first three questions, probably because these topics were developed more thoroughly in that section.

Addressing Naive Conceptions (Experimental Group)

The persistence of naive conceptions has been well documented in science education research. Two naive conceptions that are resistant to direct instruction and are of particular interest in this course are (1) the belief that plants use carbon dioxide and produce oxygen when they respire or that plants don't respire at all and (2) that there is a shunt of some type in the digestive system that sends solid waste in one direction and liquid waste in the other. Some progress was made in overcoming these naive conceptions in the experimental section, although 100% success was not achieved. Table 4 summarizes mean scores of students in the experimental section on six pretest items, three on respiration and three on the human urinary system. These are compared with mean performance on the final exam on six items about respiration and four items on the urinary system. Posttest items were more difficult than pretest items and included an essay question on the urinary system. In spite of the added difficulty of the posttest questions, student performance increased by about 60%.

Table 4

Pre- and Post-Performance (% correct) on Items Pertaining to the Urinary System and Plant Respiration (Experimental Section)

	Respiration*	**Urinary***
Pretest	41.7 (3)	46.4 (3)
Posttest	73.8 (6)	63.2 (4)

* Number of items analyzed is indicated in parentheses

Science Process Skills (Experimental and Comparison Groups)

The experimental and comparison groups were given several questions involving science process skills, all of which were taught in the comparison classes, but only some of which were taught in the experimental section. The questions were largely constructed by and graded by Dr. Roeder. The results of the posttest are summarized in Table 5.

Table 5
Process Skills
(Experimental & Comparison Sections)

	Percentage Correct		
	Experimental	Comparison 1	Comparison 2
1. Observation/Inference	92	95	95
2. Classification			
a. Consistent/Exhaustive	25	100	86
b. Hierarchical	8	77	59
3. Proper Table			
a. Missing units	88	93	82
b. Numbers out of sequence	58	84	86
c. Table made correctly	42	82	64
4. Graph Construction			
a. Metric Conversion	70	36	34
b. Labels	85	86	91
c. Scales	58	77	82
d. Choosing the right format	60	86	77
5. Experiment Design Errors			
a. Uncontrolled variable	88	91	100
b. No duplicate samples	8	68	45

Question 1, in which students distinguished between observation and inference, was probably too easy. The comparison groups significantly outperformed the experimental section on two sets of process skill questions (classification and experimental design) on which the comparison groups had received instruction and practice whereas the experimental section had none. The comparison group also did somewhat better on table and graph construction, presumably reflecting the greater emphasis given this topic in the comparison sections. Students in the experimental section had more practice with the metric system and were better able to make metric conversions.

A general conclusion is that it cannot be assumed that Liberal Studies seniors have acquired simple process skills such as constructing and interpreting graphs or developing classification systems, and that such skills will be acquired only to the extent that they are specifically taught and practiced. In the NS412C course, there is an ongoing tension (competition for time) between development of conceptual understanding and mastery of content knowledge on the one hand and acquisition of basic science skills on the other. Ideally, students would have good background knowledge of biology before coming into this course, but this clearly is not the case. Finding the most appropriate balance is a continuing challenge.

Affective Attitudes
(Experimental Group)

Upper elementary and middle school years are critical times for the development of positive attitudes toward science; yet many middle school students are turned off to science, at least in part because of ineffective instruction. With this in mind, one of the main concerns in developing this course for prospective middle school teachers is that so many of the course participants are insecure about their abilities to understand scientific phenomena and to teach it to others. Therefore, our basic task was to develop confidence in students that they are capable of doing science teaching.

Key goals were to revitalize a curiosity about science and the scientific world, and to provide experiences that would allow these future teachers to learn science in a meaningful way. The course participants were encouraged to explore, to question, and to construct knowledge of scientific phenomena in a manner that would lead to a conceptual understanding of the discipline and a willingness to further explore ideas.

Hands-on activities, group discussions, concept mapping (individual and group), and journal entries were used to facilitate knowledge development and reflective scientific thinking. A collegial, relaxed atmosphere was created in the classroom. Science was to be an enterprise of exploration and discovery, not a list of confusing terminology to be memorized or a complex system of rules to follow.

Quantitative Attitude Measures

The demographic survey showed that more than half of the NS412 students are employed for 20 or more hours per week while carrying a full course load. Many are also married and/or have children to support. Relatively few students live with their parents or in dormitories. Thus, time is a very serious constraint for these students and the NS412 course competes for their attention with many other pressing demands.

Four attitude surveys, derived from Dr. Mason's thesis research (1986) were administered at the beginning of the semester. The most interesting items from each of these were compiled into a single post-survey. In the post-survey, the wording of most items was changed to make the items correspond more closely with the specific focus of this course (for example, the word 'science' was replaced with the term 'biology' in all items).

In general, responses indicated that students in all sections developed more positive attitudes toward biology learning and teaching, more confidence about teaching biology, and a somewhat more sophisticated view of science than they had at the outset. Although the changes were nearly always positive and in the desired direction, large standard deviations preclude drawing strong conclusions. Three example items and mean student responses are shown in Table 6.

Table 6
Example Pre- and Post- Attitude Survey Items
(Experimental and Comparison Sections)

1. Pre: Learning science helps me to see how things fit together (PSS 28).
Post: By learning biology, I can see how things in nature 'fit together' (TLS 19).

2. Pre: I feel confident about teaching science content in grades 5-9 (STLS 7,8,9).
Post: I feel confident about teaching biology in middle/elementary school (TLS 17).

3. Pre: I am not looking forward to teaching science (STLS 13).
Post: I am not looking forward to teaching biology (TLS 20).

Item	Survey	Experimental	Comp 1	Comp 2
1	Pre-	4.29	3.95	4.11
	Post-	4.75	4.33	4.33
2	Pre-	3.33	3.00	2.98
	Post-	4.04	3.86	3.89
3	Pre-	-3.83	-3.86	-3.28
	Post	-4.13	-4.00	-4.00

Qualitative Attitude Measures

Students in the experimental course kept a weekly journal. Journals were graded not on content but on the quality of thoughtfulness and reflection they represented. Dr. Mason reviewed all journals and gave weekly feedback to students, guiding their attention and effort away from what they did (we didn't want progress reports) and toward what they thought and felt (that is, introspection and reflection). Below we attempt to extract the 'flavor of' and primary messages in these journals. These excerpts include comments from all but two of the students, although some more expressive students are quoted more frequently than others. Although students at first resisted writing regular journal entries, many were able to share their thoughts and feelings about biology as the semester progressed. The endeavor was difficult, as this student states:

> This week I was introduced to the concept of studying my learning processes and experience. This is what my journal entries are to reflect. The first thing that comes to my mind as I try to do this is, I don't think I know how. It's funny that I will be graduating this semester and have not yet been required to think about learning - an activity I've supposedly been doing for four years. Since this was the first week, my most profound learning experience has been [reflecting on] the importance of evaluating and thinking about learning. I now realize how crucial it is for a future teacher to examine the learning process and experiences they encounter personally in school in order to help their students. If I don't understand how I personally think and learn, how can I possibly be an effective teacher. (CC, 2/5/90)

Learning Experiences and Strategies

Negative/positive science experiences. Many students commented initially on their fear of science and their previous negative experiences with science learning. These same students later spoke of the 'fun' and 'enjoyment' associated with NS412 science. We feel quite confident that for most students, NS412 science was a positive learning experience. Four examples follow.

> I'm not a science buff, and if anything, scared of science classes because I do not do well in them, but I do try the very best I can do. (MM, 1/29/90)
>
> I enjoyed the symposium; I'm starting to question more and feel more confident. This class is becoming more fun everyday. (MM, 4/2/90)

> I am intimidated by all science. . . . In the past science has seemed confusing, tedious, boring. (JR, 1/28/90)
>
> I really enjoyed the last lab we had on flowers and identification. It was exciting to find out the different parts and sexes of the flowers. I didn't even realize flowers had different sexes. (JR, 5/7/90)

One thing I always had trouble with in other science classes (k-college) was the fact that science deals with a lot of abstract concepts and intangible ideas. (TT, 2/12/90)

Once again I must say how interesting I find this class when we are learning about our own bodies. (TT, 3/19/90)

I'm having fun growing houseplants and learning what it takes for them to stay alive and healthy looking. (TT, 4/2/90)

I always found science to be difficult and confusing (CO, 2/5/90)

I feel that because of this class, I am becoming more open-minded to science. Diseases have interested me, but overall I have dreaded taking science courses. This class is interesting and fun, so I am actually starting to enjoy science--doing instead of listening to a lecture. I feel like I am opening my mind to something I had closed it to a long time ago. (CO, 2/11/90)

Memorization vs exploration. Another aspect of learning frequently commented upon by students is the contrast between their memorization of facts in previous science classes and their active exploration of scientific phenomena in this class. A sampling of these comments follows.

I can't help but think that science without actually seeing and doing it would be both boring and confusing. (DA, 3/26/90)

There is a big difference in memorizing facts and knowing facts. There is also a big difference between looking lab answers up in the book, summarizing or paraphrasing them, . . and making the information really mean something to you. (CR, 3/5/90)

Bringing in our leaves and flowers was a great idea. It's much better to see first hand what you are studying about than to be told or see drawings in a book. This is a perfect example of how hands-on works. (LF, 5/7/90)

It is refreshing to be able to visualize processes and understand them instead of just memorizing theories and parts of a molecule. (CH, 2/26/90)

I am learning much more by using all my senses, visual, tactile and auditory. I am also retaining it for longer. I can memorize virtually anything to pass a test, but I find that once the test is over, so is my memory of the materials. (EH, 2/19/90)

> Once I have done a lab, I can usually picture what the results were. In contrast, if I read about what happened, I may not remember as much. (TK, 2/26/90)

> I find it very difficult to solve problems without using a book (or other authority) to refer to for support. It is much more stimulating and satisfying to solve something or invent something on my own; how did I get through college memorizing and repeating information from books and lectures? (TM, 2/5/90)

Concept mapping. Concept mapping is one method for making knowledge construction activities overt, and we used it several times during the semester. The technique of paper and pencil concept mapping was new to most students and, for some of them, proved to be very stimulating. Two students reported that they mapped out the entire course on their living room floor while studying for the final exam.

> I really enjoyed Wednesday's group quiz. I couldn't believe how much we discovered we knew. When we started the mapping we wondered how we were going to do this, but once we started, we couldn't stop. I discovered certain words triggered a whole other group of relationships. What's more exciting is I did this exact thing in studying for an English test and it worked. Somehow the method of visually mapping out a cluster of information aids the memory process. This device can be used in every class not just science, and I'm excited about how effective it really is. (CR, 4/23/90)

> I am continually intrigued by the concept map. We have gone from very simple ones to the create-your-own complex ones. What fascinates me is the amount of information and knowledge that can be revealed. (PC, 4/23/90)

> Doing the concept map for "life" was a great study tool. Every time we've done concept maps I thought they were very helpful. I hope that we review the semester with concept maps--soon. (JD, 5/7/90)

> When we started the concept map in class, I wasn't sure if I would be able to do it. But as we started doing the map, it got easier and easier. Once my partner and I started, we couldn't stop. We kept thinking of more and more concepts to add. I never realized how much I really knew until we did the map. It was easy to link the concepts together; I had thought that it would be tougher than it was. I think the concept map was the best way to organize all the information I have in my brain. It is finally coming together. The concept map helped to show me how I am tying everything together. (TE, 4/23/90)

Other classroom strategies. Occasionally students commented on other strategies employed in the classroom, as in the following excerpts..

> I think that working in groups like we do in class helps greatly. By working in groups, if we all see something different or the same, we can share it with each other. It also helps by seeing how other people "see" things. (TE, 3/26/90)

> Group review is more effective than individual review. . . . The knowledge you have may be incomplete, or not as clear as you would like. With group review, you have others to help. Their knowledge may be clearer than yours, and it may also "fill in the blanks" in your incomplete knowledge. Also, hearing what they have to say may help "jog" your memory and help you recall things you didn't know you knew. (HS, 4/23/90)

> One of the things I like about this class is the low-key, low-stress environment. Personally I believe I learn more and retain the knowledge better under these conditions. (PC, 4/23/90)

> The quizzes are a good idea; I seem to be learning when we grade them in class. (ET, 3/19/90)

> I just happened to think that the journal writing is another method in which I learn. It has forced me to put down on paper things that I had not thought of before, and by doing this, I was able to learn about the best ways for me to learn. (BW, 5/7/90)

Content Learning

Interesting topics. It is always refreshing to perceive students' enthusiasm about biology, particularly as it provides a striking contrast to the tentativeness and distaste they feel toward science at the beginning of the semester. When students become involved in conducting their own experiments at home, as WP describes below, we can feel hopeful that such behavior will continue beyond the scope of this class.

> I experimented with an egg in vinegar, accidentally broke the first one, but allowed it to remain in the vinegar to see what would happen. The second one remained whole, and decalcified (white bubbles on top) and was very soft. The first one's white and yolk leaked out and it looked like an embryo. The shell completely collapsed. (WP, 2/26/90)

Other students also gave signs of becoming engaged in active science learning.

> Watching the change of the worm to a beetle was really interesting. We saw the worms in different stages, changing colors, growing, and finally the bug. An amazing transformation . . . . (CC, 4/23/90)

> I was excited to see that we finally got a beetle. At first these observations were gross to me, but now they have moved from gross to interesting. (TK, 4/23/90)

> What made the basic processes lab meaningful to me was being able to transfer and utilize and apply knowledge I had previously gained to a new application. That is what learning is all about. (PC, 2/26/90)

Naive conceptions. Relatively few students commented on their naive conceptions and how they had changed. This is interesting, given the supposedly widespread occurrence of naive conceptions. Perhaps students are reticent to talk about their alternative ideas.

> I knew before coming into class today that in our bodies, we take in oxygen and give off carbon dioxide, but I didn't realize that ordinary air doesn't really have carbon dioxide in it--or very little. I never really thought about it because we breathe in and out all of the time, so I assumed it was all the same air. (TE, 1/29/90)

> I thought the water flowed throughout the plant in the same way blood goes through our body. I didn't realize it was controlled by the stoma. I broke open the celery--looked as if there were straws in the celery. (CP, 3/5/90)

Conceptual change. Only one student observed how difficult it can be to displace past knowledge with new information (below). The infrequency with which students commented on this process is noteworthy, especially in light of our current research project on conceptual change.

> Even with the new information, I still tended to recall and remember my "past knowledge". . . . It was hard displacing "past information: or "past knowledge", and replacing it with new knowledge. (HS, 2/7/90)

Process Skills

Questioning. A major theme in students' comments is their new view of the world around them. They begin to ask questions and to analyze events. It is as if a heavy lid had fallen on their curiosity at some time in the past, and we managed to pry that lid off.

> I feel I have been opening my mind up to a new angle of thought on all information that confronts me anywhere. I've learned to think scientifically about all of the things around me all of the time--even if it's only the rate at which water flows out of a water fountain. I find myself wondering how many

> moles of water are in a small stream and what kinds of bonds are being broken and formed as the water flows from the pipe into the open atmosphere. (MA, 3/12/90)

> Science is starting to make me question everything. Whenever I am cooking something, I question the baking process. (AB, 3/26/90)

> I don't especially like to think that I don't even question life happening around me constantly, but it's like I'm beginning to throughout this class. I'm noticing processes and thinking "how" or "why" things are or came to be. (RA, 3/5/90)

> Planting these beans in soil and putting them in a moist environment made me think about if beans really do need soil. It also made me question the purpose of soil, other than nutrients and grounding for the plant's roots. It also made me question how a bean sprouts. Do they sprout from the middle or the side? (DD, 3/19/90)

> I wonder what plant hormones are and if they don't have glands, where do plants get their hormones? (LF, 3/26/90)

> I thought it would be hard to think of science during spring break , but I was wrong. I forgot about the inquisitive nature of my kids. (BW, 4/16/90)

Wonder. Wonder and curiosity are the benefits of a questioning mind. The excitement of discovery in these excerpts seems especially poignant.

> Walking through the exhibit [during the Natural Science Museum field trip] made the reading come alive. It seems the more you view a topic and the more perspectives and different views you take of a topic, the more accessible it becomes. (CR, 2/19/90)

> Last week I felt like a kid learning so many different things. I was actually at the museum discovering something that is practically in my backyard but that I was unaware of. (TR, 2/19/90)

Observing. Students in NS412 seem to see the world in new ways:

> I have noticed an exciting change in my outlook, that I believe is a direct outcome of this class. Lately I have been much more observant of things around me. I think the fact that the labs concentrate on observation skills has run over into everyday life. Instead of just walking across campus without looking at anything, I now find myself noticing birds, plants, trees, and the way people behave.

> The methods of deductive reasoning, observation, etc. are not only useful in a science class. They are useful in everyday relationships.

> Observation skills need to be developed because they are necessary to life--not just science. My observation skills need a lot of work, but I am pleased to report that as a direct result of this class, I am trying harder to observe and thus am becoming very aware of the world around me, which helps me enjoy it more.

> While jogging, I started looking around at all of the plants, amazing; the review helped; I observed surface tension in champagne (KS, 2/26/90)

Negative Response to Hands-On Science

Two students wrote very little in their journals. A third student was put off by the structure of the class and had a negative attitude overall, as suggested by the following brief comments:

> I learn best when I either read or hear a lecture. . . . I learn best alone. (ND, 2/26/90)

> Biology is too unpredictable; I like the neatness of computer science. (ND, 3/26/90)

This student had completed the largest number of science courses, and the pace of this course was slow for her. She could have used her skills to lead her group, but she demonstrated an inability and/or unwillingness to share her knowledge. She appeared to have difficulty relating her book learning to hands-on experiences.

End of Course Reflections

The students below seem to speak for many in the class as they look back and describe how they were affected by NS412 this semester, coming to enjoy science and to feel good about 'doing it'.

> Overall, this has been a great class. As I'm sure you know, not very many people like science. As a result, many elementary school teachers kind of skip over or barely touch on the subject of science. This is partly because they don't enjoy science but mostly because they feel inadequate to teach it. However, I think you have made a tremendous difference to many people in our class. You certainly did to me. When I say "you" I mean both of you. Somewhere along the way, we learned a great deal of science and found ourselves liking it. My favorite part of this class was not that it taught me so many new things. I had already learned much of the material in biology, genetics, health, and nutrition at Grossmont College before I transferred. However, I forgot most of what I learned in those classes

> because I was so busy trying to read, memorize and cram for tests that I never got to let any of it sink in or tie together. (TK, 5/7/90)

> Since this is the final entry, I would like to begin by saying this has been an excellent learning process. Writing about your learning experiences really helps you figure out how and why you learn certain things. Throughout the semester I have made some very interesting discoveries about science, learning, and myself. I can honestly say for the first time in my schooling I have truly enjoyed science. In the syllabus one of the goals was for us to "rediscover our natural, latent curiosity and to apply it, in a little more disciplined manner than that typically used by children to explore the nature of life and living things." I feel this goal was successfully met, which is why I enjoyed science. The experiments with their hands-on approach greatly piqued my curiosity. The experiments were fun and of interest. I found the more we experimented the more I began to question everything around me. The process of questioning was not limited to the laboratory. I began to open my eyes to the world surrounding me and discovered little things I had never noticed in all my 24 years. I no longer walk absentmindedly through campus. Now I observe how leaves are attached to plants and trees, and ask myself why they are attached that way. Basically I observe and question more about everything. I also discovered that questions I thought were impossible to answer are fairly simple if you apply common sense and probe deep enough. I used to think that science was for nerds and really had no importance in my life. This attitude really hindered my learning. But this class has shown me that science is for everyone and plays a role in every aspect of life. Instead of feeling like a nerd for studying science, I feel enlightened--as though science is a wonderful secret I've been let in on. This course has also taught me a lot about learning in general. I had never even thought about how I learned, somehow I just did it. Through this course I have learned that everyone has a different way of learning, but everyone seems to benefit from the hands-on approach. When you actively learn something, it is much easier to remember than just reading. This course has also taught me something about myself. That is that I can be scientific and can be successful if I apply myself. This is important for a future teacher; now that I enjoy and feel successful in science, I can teach it to my students. Thank you for showing us that science is fascinating as well as fun; I believe all of us will be better teachers for this enlightenment. (CR, 5/7/90)

Interpreting Student Journals. In some ways, student journals provide more insights into what the students are thinking than any other measures we used. On the other hand, it is well to keep in mind that students knew their journals were being read by the instructors and also that they were being graded (albeit on the 'quality of students' reflectivity', not on specific content or attitude). Since students are understandably anxious to please their instructors in most cases, we can only wonder how much the content might differ if students were keeping notes for their eyes only. At a minimum, it seems fair to conclude that many students acquired our enthusiasm. Whether the course experience produces any long term impact remains to be seen. (We are planning to perform a retention study in spring 1991.)

Discussion and Conclusions

Positive Outcomes

Of the many aims of the NS412C course on process and inquiry in biology, the affective goals are perhaps the best realized. Many students come in the door fearful of science and expressing distaste for science. However, they gradually become involved in doing science at a modest level, and for the most part they thoroughly enjoy it. Many students also have a limited view of the nature of science learning--namely, they equate science learning with rote mastery of facts. In NS412C they learn that there are other ways to achieve learning and that hands-on experience, collaboration, and discussion can be much more satisfying than memorization and can also lead to deeper understanding. Students make significant gains in their knowledge of some topics in biology and they improve their basic science skills, but in both areas, not as much as we would like.

Negative Outcomes

The increased emphasis on conceptual understanding in the experimental course was achieved by decreased emphasis elsewhere -- in particular, in the development of basic science skills. Pre- and posttest performance demonstrated that the comparison groups learned science process skills that had not been emphasized in the experimental section, particularly with respect to use of duplicate samples, graphing, and developing classification systems. Which is more important? Arguments for emphasizing process skills include: 1) It is impossible in this course to cover all of the biology content that would be needed by future teachers who might teach any grade between kindergarten and eighth or ninth grade. 2) Thus students should be armed with an assortment of experimental techniques so that they can generate experiments on their own. On the other hand, these students may lack the conceptual background to apply science process skills independently. Arnold Aarons argues (personal communication) that it is better to give these students a set of well developed experiments that they understand well and can reliably use.

Materials and Strategies Developed

This project laid a foundation for conceptually oriented teaching of NS412C for middle school teachers, with incorporation of cognitive science principles, in terms of

(a) design of content materials incorporating cognitive and constructivist theory,

(b) development of instructional strategies for promoting cognitive change, and

(c) development of survey and test instruments for measuring cognitive change.

Instructional Design

Instruction was designed for several course topics that we perceived to be important for middle school science teaching but not necessarily relevant for teaching K-6. These include laboratories on molecular structure, diffusion and other basic biological processes, and the movement of chromosomes in mitosis and meiosis (see Appendix C.3 for examples). Strategies were developed for making these complex ideas accessible to middle school children. Examples include:

- a pattern matching task with organic molecules in which students, with no previous instruction, sort molecules and discover the four major classes of molecules based on similarities and differences in molecular structure;
- observations of diffusing molecules in gases (aromatic molecules crossing the room), liquids (coffee and sugar solutions), and solids (paper chromatography with colored inks);
- use of familiar shapes (knife, fork and spoon) to represent chromosomes and to work through the processes of mitosis and meiosis.

Students were also introduced to local resources available to teachers, especially the science museums. For example, students conducted simulated ecology field research at the Natural History Museum, using the desert exhibit and curriculum materials developed by Dr. Judy Diamond (Diamond, Bond, & Hirumi, 1989).

Strategies

One strategy for promoting both conceptual development and lifelong learning of biology is the daily discussion and interpretation of newspaper articles about biology topics. A strategy for reinforcing conceptual knowledge is to go around the room asking each student to say one sentence about a particular topic (such as respiration or diffusion) that no one else in the class has said (this brief exercise can be readily combined with taking roll). Frequent quizzes with in-class grading and discussion encourage students to think about previous classes and to invest at least some mental energy in trying to recall and make sense of them. Discussing a topic and making predictions about the outcome of an experiment (as modeled by Jim Minstrell (1990) in videotapes prepared for NSF) is an effective means for engaging students in thinking about the topic being studied.

Instruments

The final survey instrument for measuring attitudes toward biology learning that was developed and tested during this pilot study will be useful in subsequent studies. Likewise the pre- and posttests developed and piloted during this project will be useful in future studies (including the Goldberg and Fisher (1991) TPE-funded project now underway).

Key Challenges for the Future

We identified four areas or foci that deserve further exploration and demand special attention in efforts to change the ways we do business in the classroom.

• Acquiring new learning habits. Most of our students appear to rely heavily on rote learning strategies and probably have done so for most of their sixteen or more years in school. In this project we tried not only to teach biology, but also to change students' 'worldviews' and 'basic values' regarding what is important in education. That is, we tried to promote respect for and development of habits of meaningful learning. The challenge of producing long-term change in such basic attitudes and modalities should not be underestimated. This is a very difficult task.

• Conflict between 'Learning About' and 'Learning How To'. Learning by observation and experimentation is relatively slow and requires more skill and self-confidence than does looking up an 'answer' in a book. The two sources of information (laboratory and textbook), rather than reinforcing one another, actually produce a learning conflict. We find we can engage students more fully in learning through experience if we do not use a text. This is unfortunate.

• Depth/breadth tradeoff. The depth/breadth issue is paramount in biology, where the volume of what is known (and therefore available to be taught) increases daily. The depth approach seems to produce more meaningful learning, but there are no easy algorithms for deciding what to include and what to omit. This is perhaps the greatest challenge for the constructivist approach to teaching.

• Retooling teachers. Training instructors to teach in new ways is a non-trivial problem. Since under pressure teachers tend to revert to the methods they know best, it probably takes at least five years of hard work and committed practice to make a transition from 'teacher-centered, coverage-oriented teaching' to 'student-centered, depth-oriented' teaching. The process also requires a great deal of reflection and support. These are key considerations for future planning.

References

Abruscato, J., Fossaceca, J. W., Hassard, J., & Peck, D. (1980). *Holt Elementary Science*. New York: Holt, Rinehart, & Winston.

Alberti, D., Davitt, R. J., Ferguson, T. A., & Repass, S. O. (1976). *Elementary Science Study*. New York: McGraw Hill.

Anderson, J. R. (1983). *The architecture of cognition*. Cambridge, MA: Harvard University Press.

Diamond, J., Bond, A., & Hirumi, A. (1989). Desert explorations: A videodisc exhibit designed for flexibility. *Curator, 32* (3), 161-173.

Goldberg, F. & Fisher, K. M. (1991). *Making the invisible visible: A learning environment to enhance conceptual understanding in physics and biology*. National Science Foundation Grant # TPE-9053803.

Mason, C. L. (1986). Student attitudes toward science and science related careers: An investigation of the efficacy of a high school biology teachers' intervention program. *Dissertation Abstracts International*, **47**, (6) 2105A.

Minstrell, J. (1990, April). *A Teaching System for Diagnosing Student Conceptions and Prescribing Relevant Instruction.* Presented at the Annual Meeting of the American Educational Research Association, Boston.

Scardemalia, M., Bereiter, C., McLean, R. S., Swallow, J., & Woodruff, E. (1989). Computer supported intentional learning environments. *Journal of Educational Computing Research*, **5** (1), 51-66.

Stepans, J. (1985). Biology in Elementary Schools: Children's Conceptions of 'Life'.*The American Biology Teacher*, **47** (4), 222-225.

PUBLICATIONS ACKNOWLEDGING THIS PROJECT

Sowder, J., Sowder, L., & Bezuk, N. (In press) Using principles from cognitive psychology to guide rational number instruction for prospective teachers. In T. Carpenter & E. Fennema (Eds.) *Learning, Teaching, and Assessing Rational Number Concepts: Multiple Research Perspectives.*

Mason, C. L. (1991). *Concept mapping: A tool to develop reflective science instruction.* Manuscript submitted for publication.

Mason, C. L., Diehl, P., Peirce, A., Dessel, N., & Johnston, R. (1991). *Preparing science teachers for the 21st century.* In process.

Following completion of a spring 1991 retention study on the Life Science section of this project, a publication will be prepared incorporating the retention study results with those described herein.

PRESENTATIONS ACKNOWLEDGING THIS PROJECT

Bezuk, N., Sowder, J., & Sowder, L. (1990, July) *A cognitive approach to instruction of prospective teachers.* Poster Session presented the Fourteenth International Group for the Psychology of Mathematics Education Conference, Mexico.

Fisher, K. (1989, October). *Promoting habits of meaningful learning.* Feature presentation at the national convention of the National Association of Biology Teachers, San Diego, CA.

Flores, A., & McLeod, D. (1990, November). Calculus for middle school teachers using computers and graphing calculators. *Proceedings of the Third Annual International Conference on Technology in Collegiate Mathematics.* Columbus: Addison-Wesley.

Mason, C. L. (1990, March). *Conceptualizing Science: Activities designed to help students see the big picture.* Workshop presented at the annual meeting of the San Diego Science Educators Association, San Diego, CA.

Mason, C. L. (1990, April). *Science teaching and learning: Using concept mapping to develop reflective practitioners.* Paper presented at the national meeting of the National Association for Research in Science Teaching, Atlanta, GA.

Mason, C. L. (1990, November). *Concept mapping: A tool to develop reflective practitioners.* Paper presented at the area convention of the National Science Teachers Association, Long Beach, CA.

Mason, C. L. (1990, November). *The world of middle school biology: Focus on it.* Paper presented at the annual meeting of the National Association of Biology Teachers, Houston, TX.

Mason, C. L. & Fisher, K. (1990, March). *Middle school biology: A cognitive approach.* Workshop presented at the annual meeting of the San Diego Science Educators Association, San Diego, CA.

Sowder, J. (April, 1990). *Using principles from cognitive psychology to guide rational number instruction for prospective teachers.* Presentation at the Research Presession for the Annual Meeting of the National Council of Teachers of Mathematics, Salt Lake City.

APPENDIX A.1

CALCULATORS IN CALCULUS: THAT'S THE LIMIT

Part 1: Preliminary Explorations with Sequences

The following sequences can be explored by repeatedly pushing a single key after a number has been entered, or by using the number repeatedly as constant factor.

Activity 1. Constant factor.

Most calculators have the capability to use the same number as a constant factor (or addend or whatever). Some have a special key for that, others do not need one. The following examples will be illustrated for a calculator that has a constant factor capability without a special key (Sharp EL-506S) and for a calculator that remembers the last set of instructions (Casio fx-7000G). They are given to convey the iterative process to calculate the sequences. The keys to be pushed may vary with other types of calculators.

a) Push the following keys in your calculator.
Sharp EL-506S

$2\ \boxed{\times}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ ....

Casio Fx 7000G

$2\ \boxed{\text{EXE}}\ \ \boxed{\times}\ \boxed{\text{ANS}}\ \boxed{\text{EXE}}\ \ \boxed{\text{EXE}}\ \boxed{\text{EXE}}\ \boxed{\text{EXE}}$ . . .

Write the number on the display after pushing each set of keys:

$S_1 = 2$
$S_2 = 4$
$S_3 = 8$
$S_4 = 16$
$S_5 = 32$
$S_6 = 64$
$S_7 = 128$
$S_8 = 256$
$S_9 = 512$
$S_{10} = 1024$
$S_{11} = 2048$
$S_{12} = 4096$
$S_{13} = 8192$
$S_{14} = 16384$
$S_{15} = 32768$
$S_{16} = 65536$
. . .

These numbers appear in the famous problem of the chessboard, or in the number of sheets when you fold a paper in half repeatedly.

b) Start with a different number. Write down your results.

What happens if you start with 1? _________ Why?

What happens if you start with a number between 0 and 1? _____

Try 0.1
Sharp EL-506S

$0.1\ \boxed{\times}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}\ \boxed{=}$ ....

Casio Fx 7000G

$0.1\ \boxed{\text{EXE}}\ \ \boxed{\times}\ \boxed{\text{ANS}}\ \boxed{\text{EXE}}\ \ \boxed{\text{EXE}}\ \boxed{\text{EXE}}\ \boxed{\text{EXE}}$ . . .

Write down the number on the display after each execution
0.1
0.01
0.001
0.0001
. . .

What number do these numbers approach? _____ Notice that we can come as close to 0 as we want. Zero is the limit of this sequence.

Use other numbers to start with. Try 0.9
Sharp EL-506S

0.9 $\boxed{\times}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ $\boxed{=}$ ....

Casio Fx 7000G

0.9 $\boxed{\text{EXE}}$ $\boxed{\times}$ $\boxed{\text{ANS}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

Write down the number on the display after each execution
0.9
0.81
0.729
0.6561
0.59049
. . .

What number do these numbers approach? _____

What keys do you have to push to obtain the following sequences?

$0.99, 0.99^2, 0.99^3, 0.99^4, \ldots$

$1.00001, 1.00001^2, 1.00001^3, 1.00001^4, \ldots$

What happens if you start with 0? ________

What happens if you start with a negative number? Try first a number between -1 and 0. Try a number less than -1.

Represent the sequence by algebraic expressions, assuming the first term is a.
What can you say about the sequence
$a,\ a^2,\ a^3,\ a^4, \ldots \qquad (0 \le a < 1)$

What can you say about the sequence
$a,\ a^2,\ a^3,\ a^4, \ldots \qquad (1 < a)$

Activity 2
Push the following keys:
Sharp EL-506S

2 $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ ....

Casio Fx 7000G

2 $\boxed{\text{EXE}}$ $\boxed{x^2}$ $\boxed{\text{ANS}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

What numbers appear on the display? Write them down.

2, 4, 16, 256, 65536, . . .

What is the relationship between these numbers and those in the first example of Activity 1?
How quickly do you reach numbers bigger than the calculator is able to deal with? __________
Each number is the square of the previous one. Represent this sequence using exponents.
What is the relationship of the exponents and the numbers in the first example of Activity 1?

Start with a different number. Choose a number that is very close to 1, for example 1.000001
What are the numbers of the sequence this time?

1.000001 $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ ....

Sharp EL-506S
Casio Fx 7000G

1.000001 $\boxed{\text{EXE}}$ $\boxed{x^2}$ $\boxed{\text{ANS}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

1.000001
1.000002
1.000004
1.000008
1.000016
1.000032
. . .

Now choose a number close to 1 but smaller, for example, 0.999999.
What happens with the numbers of the sequence in this case?
Sharp EL-506S

0.999999 $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ $\boxed{x^2}$ ....

Casio Fx 7000G

0.999999 $\boxed{\text{EXE}}$ $\boxed{x^2}$ $\boxed{\text{ANS}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

0.999999
0.999998
0.999996
0.999992
0.999984

Represent this type of sequence with an algebraic expression. Use exponents.

The sequence can also be defined by

$S_n = S_{n-1} * S_{n-1}$ $S_1 = 0.99999$

or, in general,

$S_n = S_{n-1} * S_{n-1}$ $S_1 = a \quad (0 \leq a < 1)$.

What relationship is there between this sequence and the one on Activity 1? ______ How much faster does this subsequence converge?

Activity 3
Push the following keys:
Sharp EL-506S

2 $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ ....

Casio Fx 7000G

2 $\boxed{\text{EXE}}$ $\boxed{\sqrt{}}$ $\boxed{\text{ANS}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

Observe the numbers that appear on the display after each execution.
2
1.4142 . . .
1.18 . . .
1.09 . . .
1.04 . . .
1.02 . . .
What number do they approach?
The sequence you are exploring is

$2, \quad \sqrt{2}, \quad \sqrt{\sqrt{2}}, \quad \sqrt{\sqrt{\sqrt{2}}}, \ \ldots$

Express this sequence using fractional exponents (remember that $\sqrt{x} = x^{1/2}$)

What happens if you start with a much bigger number, for example, 1000000?
Sharp EL-506S

1000000 $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ $\boxed{\sqrt{}}$ ....

Casio Fx 7000G

1000000 $\boxed{\text{EXE}}$ $\boxed{\sqrt{}}$ $\boxed{\text{ANS}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

Write down the numbers.
1000000
1000
31.62 . . .
5.62 . . .
2.37 . . .
1.53 . . .
Do they approach a number? ________
What happens if you start with 1? _______
Or with 0? ________

What happens if you start with a number between 0 and 1, for example 0.000001?
Sharp EL-506S

0.000001 $\boxed{\sqrt{\ }}$ $\boxed{\sqrt{\ }}$ $\boxed{\sqrt{\ }}$ $\boxed{\sqrt{\ }}$ $\boxed{\sqrt{\ }}$ ....

Casio Fx 7000G

0.000001 $\boxed{\text{EXE}}$ $\boxed{\sqrt{\ }}$ $\boxed{\text{ANS}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

0.000001
0.001
0.0316 . . .
0.177 . . .
0.42 . . .
0.64 . . .
0.80 . . .
0.89 . . .

Write the terms of this sequence in algebraic notation.

Part 2: Exploring More Sequences

In the following examples, the terms of the sequences are not obtained with a single key push. Students have to push a series of keys in a certain order to obtain the successive terms. The sequences of key strokes shown work with the mentioned scientific calculators. It is shown to convey the iterative process to obtain the sequence and to suggest how the sequence of key punches can be translated into a program. The actual sequence of keys punched can vary for each type of calculator.

Activity 4

Compute the following terms of this sequence.
$\sqrt{2}$, $\sqrt{(2\sqrt{2})}$, $\sqrt{(2\sqrt{(2\sqrt{2})})}$, . . .
To obtain these on the calculator push the following keys and observe the numbers on the display after you push each set of keys.
Sharp EL-506S

2 $\boxed{\sqrt{\ }}$ $\boxed{\times}$ 2 $\boxed{=}$ $\boxed{\sqrt{\ }}$ $\boxed{\times}$ 2 $\boxed{=}$ $\boxed{\sqrt{\ }}$ ....

Casio Fx 7000G

$\boxed{\sqrt{\ }}$ 2 $\boxed{\text{EXE}}$ $\boxed{\sqrt{\ }}$ $\boxed{(}$ 2 $\boxed{\times}$ $\boxed{\text{ANS}}$ $\boxed{)}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

1.4142 . . .
1.6817 . . .
1.8340 . . .
1.9152 . . .
1.9571 . . .
1.9784 . . .

What value do these numbers approach? ______

Express the sequence using exponents.
$\sqrt{2} = 2^{1/2}$
$\sqrt{(2\sqrt{2})} = 2^{3/4}$
$\sqrt{2\sqrt{(2\sqrt{2})}} = 2^{7/8}$
Find the general term.

Start with a different number.
Write the sequence in algebraic terms, using exponents, assuming the starting number is $\sqrt{M}$

$\sqrt{M}$, $\sqrt{(M*\sqrt{M})}$, $\sqrt{(M*\sqrt{(M*\sqrt{M})})}$, ...

Activity 5
Use the calculator to compute the terms of the following sequence.
$\sqrt{2}$, $\sqrt{(2+\sqrt{2})}$, $\sqrt{(2+\sqrt{(2+\sqrt{2})})}$, . . .

You can do this by pushing the keys as follows:
Sharp EL-506S

2 $\boxed{\sqrt{\ }}$ $\boxed{+}$ 2 $\boxed{=}$ $\boxed{\sqrt{\ }}$ $\boxed{+}$ 2 $\boxed{=}$ $\boxed{\sqrt{\ }}$ ....

Casio Fx 7000G

$\boxed{\sqrt{\ }}$ 2 $\boxed{\text{EXE}}$ $\boxed{\sqrt{\ }}$ $\boxed{(}$ 2 $\boxed{+}$ $\boxed{\text{ANS}}$ $\boxed{)}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

Observe the numbers on the display after each execution
1.4142 . . .
1.8477 . . .
1.9615 . . .
1.9903 . . .
1.9975 . . .
1.9993 . . .

What number L do these numbers approach?
What is the relationship between L and L^2? ______ Write an equation. ___________ What is the value of the limit? _________

Compute the terms of the following sequence:
$\sqrt{5}$, $\sqrt{(5+\sqrt{5})}$, $\sqrt{(5+\sqrt{(5+\sqrt{5})})}$, . . .

Observe the decimal expansions of L and L^2 on the display.
2.236 . . .
2.689 . . .
2.773 . . .
2.788 . . .
2.7907 . . .
2.7911 . . .
2.79126 . . .
2.791284 . . .
2.791287 . . .

What is the relationship between L and L^2? Write an equation that describes this relationship. What is the exact value of L?

Activity 6. The golden section.
Compute the following sequence:

$\sqrt{1}$, $\sqrt{(1+\sqrt{1})}$, $\sqrt{(1+\sqrt{(1+\sqrt{1})})}$, . . .
You can do this by pushing the keys
Sharp EL-506S

$1\ \boxed{\sqrt{\ }}\quad \boxed{+}\ 1\ \boxed{=}\ \boxed{\sqrt{\ }}\quad \boxed{+}\ 1\ \boxed{=}\ \boxed{\sqrt{\ }}\quad \ldots\ldots\ldots\ldots$

Casio Fx 7000G

$1\ \boxed{\text{EXE}}\quad \boxed{\sqrt{\ }}\ \boxed{(}\ 1\ \boxed{+}\ \boxed{\text{ANS}}\ \boxed{)}\ \boxed{\text{EXE}}\quad \boxed{\text{EXE}}\ \boxed{\text{EXE}}\quad \ldots$

Observe the numbers on the display after each execution.
1
1.4142 . . .
1.553 . . .
1.598 . . .
1.611 . . .
1.616 . . .
1.617 . . .
1.6178 . . .
1.6179 . . .
1.6180 . . .
1.61802 . . .
1.61803 . . .
1.618033 . . .

Continue with the process until the numbers on the display do not change. Write down the result, r = __________
What is the relationship between r and r^2? (Observe the numbers after the decimal point for each one. Express this relationship by an equation:

$r + 1 = r^2$

The sequence can also be defined by
$S_n = \sqrt{(1+ S_{n-1})}$ $\qquad S_1 = 1$

Activity 7
Compute the following sequence.
1, 1 + 1, 1 + 1/ (1+1), 1 + 1/ (1+1/(1+1)), . . .
One way is to calculate the terms is to push the following key sequence:
Sharp EL-506S

$1\ \boxed{1/x}\ \boxed{+}\ 1\ \boxed{=}\quad \boxed{1/x}\ \boxed{+}\ 1\ \boxed{=}\quad \boxed{1/x}\ \boxed{+}\ 1\ \boxed{=}\ \ldots$

Casio Fx 7000G

$1\ \boxed{\text{EXE}}\quad \boxed{+}\ \boxed{\text{ANS}}\ \boxed{x^{-1}}\ \boxed{\text{EXE}}\quad \boxed{\text{EXE}}\ \boxed{\text{EXE}}\quad \ldots$

If the calculator does not have a key for the reciprocal, the memory key can be used, for example:

$$1\ \boxed{x\to M}\ 1\ \boxed{\div}\ \boxed{RM}\ \boxed{+}\ 1\ \boxed{=}\quad \boxed{x\to M}\ 1\ \boxed{\div}\ \boxed{RM}\ \boxed{+}\ 1\ \boxed{=}\ \ldots$$

Write down the numbers on the display after each execution.
1
2
1.5
1.6666666
1.6
1.625
1.6153 . . .
1.6190 . . .
1.6176 . . .
1.6181 . . .

Continue the process until the numbers on the display do not change.
Write down the result r =
What is the relationship between r and 1/r? Observe their decimal expansions. Express this relationship with an equation.
$r = 1 + 1/r$
What is the relationship of this limit with the one in the previous activity? _______ Why?

Observe that each term is obtained by adding 1 to the reciprocal of the previous term, or $S_n = 1 + 1/S_{n-1}$
thus the sequence is
$S_1 = 1$
$S_2 = 1 + 1/1$

$$S_3 = 1 + \frac{1}{1 + 1/1}$$

$$S_4 = 1 + \frac{1}{1 + \frac{1}{1 + 1/1}}$$

$$S_5 = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + 1/1}}}$$

This suggests expressing the limit of the as a continued fraction:

$$L = 1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \frac{1}{1 + \ldots}}}}$$

Activity 8
The limit of the two previous sequences is known as the golden ratio; it is the positive root of the equation, and its exact value is $(1+\sqrt{5})/2$.
We can get approximations to this value using the Fibonacci numbers:
1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, 144, 233, 377, 610, . . .

Each term after the first two is obtained by adding the two previous numbers $F_n = F_{n-1} + F_{n-2}$

Compute and write the next terms of the Fibonacci sequence (until you have 20 terms).
To approximate the golden ratio, divide each term of the sequence by the preceding term:

1/1, 2/1, 3/2, 5/3, 8/5, 13/8, 21/13, 34/21, 55/34, . . .

1
2
1.5
1.6666666
1.6
1.625
1.6153 . . .
1.6190 . . .
1.6176 . . .
1.6181 . . .

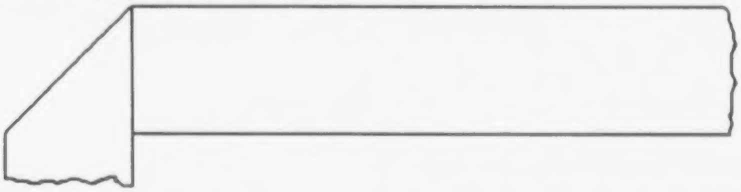
or in general

$S_n = F_n / F_{n-1}$

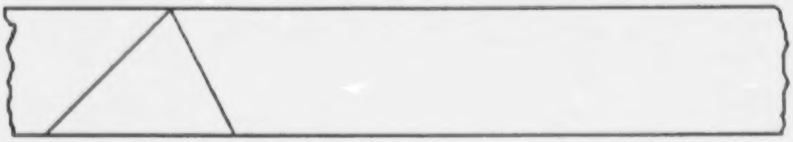
where F_n is the *n*th term of the Fibonacci sequence

Activity 9 Folding a strip of paper
Cut a strip of paper 5 cm wide and about 50 cm long (adding machine paper works very well).

Fold a crease close to one end at an arbitrary angle. (It can be any angle, but here we will illustrate the procedure for an angle A less than 90°)



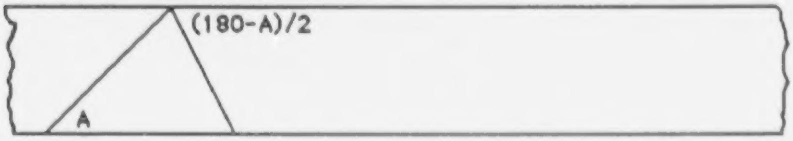
By folding, bisect the angle formed by this crease and the edge of the strip. Bisect the obtuse angle formed by this new crease and the edge of the strip.



Continue this procedure several times.
What shapes do the successive creases form?

No matter what initial angle you take for your first crease, the successive angles always approach 60° in measure.

Folding the strip of paper as indicated will form an obtuse angle that is (180 - A), bisecting it will give an angle B = (180 - A) /2, as indicated.



The following angle will be (180 - B)/2. So that in general
$S_n = (180 - S_{n-1})/2 \quad S_1 = A \quad (0 < A < 180)$

Starting with an angle of A = 28° the sequence of key strokes would be:

Sharp EL-506S

2 8 $\boxed{+/-}$ $\boxed{+}$ 180 $\boxed{=}$ $\boxed{\div}$ 2 $\boxed{=}$ $\boxed{+/-}$ $\boxed{+}$ 180 $\boxed{=}$ $\boxed{\div}$ 2 $\boxed{=}$ . . .

Casio Fx 7000G

28 $\boxed{\text{EXE}}$ $\boxed{(}$ 180 $\boxed{-}$ $\boxed{\text{ANS}}$ $\boxed{)}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ $\boxed{\text{EXE}}$ . . .

The angles would be:
28, 76, 52, 64, 58, 61, 59.5, 60.25, . . .

Notice that the numbers are alternately bigger and smaller than 60, and that the difference from 60 is halved with each successive term.

Activity 10

Remember that n! = n x (n-1)!, and 0! = 1
$S_n = 1/n!$
For this sequence, the key stroke sequence can be
Sharp EL-506S

1 $\boxed{\div}$ 2 $\boxed{=}$ $\boxed{\div}$ 3 $\boxed{=}$ $\boxed{\div}$ 4 $\boxed{=}$ . . .

Casio Fx 7000G

1 $\boxed{\div}$ 2 $\boxed{\text{EXE}}$ $\boxed{\div}$ 3 $\boxed{\text{EXE}}$ $\boxed{\div}$ 4 $\boxed{\text{EXE}}$ . . .

Write down the terms:
1
0.5
0.166666666
0.041666666
0.008333333
0.001388888
0.000198412
0.000024801
0.000002755
0.000000275
0.000000025
0.000000002
. . .

Part 3

The following sequences are of a special kind called series. The *n*th term of the sequence S_n is made of a sum of *n* addends.

$S_1 = a_1$
$S_2 = a_1 + a_2$
$S_3 = a_1 + a_2 + a_3$
. . .
$S_n = a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

The notation $\sum_{k=1}^{n} a_k$

is used as a short hand for $a_1 + a_2 + a_3 + a_4 + \ldots + a_n$

Activity 11

Explore the following sequence with your calculator.

$S_1 = 1/2$
$S_2 = 1/2 + 1/4$
$S_3 = 1/2 + 1/4 + 1/8$
$S_4 = 1/2 + 1/4 + 1/8 + 1/16$
. . .
$S_n = 1/2 + 1/4 + 1/8 + \ldots + 1/2^n$

That is,

$$S_n = \sum_{k=1}^{n} 1/2^k$$

The key stroke sequence with a calculator with memory key and reciprocal key would be:
Sharp EL-506S

$2\ \boxed{1/x}\ \boxed{M+}\ \boxed{RM}\quad 4\ \boxed{1/x}\ \boxed{M+}\ \boxed{RM}\quad 8\ \boxed{1/x}\ \boxed{M+}\ \boxed{RM}\ \ldots$

Casio Fx 7000G

$1\ \boxed{\div}\ 2\ \boxed{EXE}\quad \boxed{+}\ 1\ \boxed{\div}\ 4\ \boxed{EXE}\quad \boxed{+}\ 1\ \boxed{\div}\ 8\ \boxed{EXE}\ \ldots$

Write down the terms of the sequence

$S_1 = 0.5$
$S_2 = 0.75$
$S_3 = 0.875$
$S_4 = 0.9375$
$S_5 = 0.96875$
$S_6 = 0.984375$
$S_7 = 0.9921875$
$S_8 = 0.99609375$
$S_9 = 0.998046875$
$S_{10} = 0.999023437$
. . .

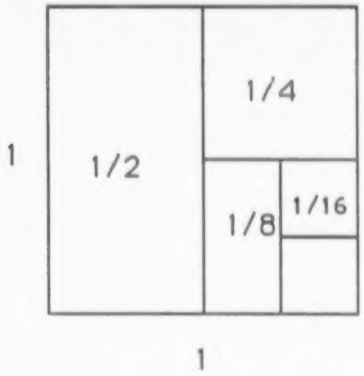
An alternative way to obtain the sequence is to push the following keys
Casio Fx 7000G

$1\ \boxed{\rightarrow}\ \boxed{\text{ALPHA}}\ \boxed{\text{R}}\ \boxed{:}\ 0\ \boxed{\rightarrow}\ \boxed{\text{ALPHA}}\ \boxed{\text{S}}\ \boxed{\text{EXE}}$

and these keys (we will omit the ALPHA for letters, but remember to push the ALPHA key before them)
Casio Fx 7000G

$\boxed{\text{R}}\ \boxed{\div}\ 2\ \boxed{\rightarrow}\ \boxed{\text{R}}\ \boxed{:}\ \boxed{\text{R}}\ \boxed{+}\ \boxed{\text{S}}\ \boxed{\rightarrow}\ \boxed{\text{S}}\ \boxed{\text{EXE}}$

To obtain the terms of the sequence just keep pushing the EXE key.
This series can also be interpreted geometrically:

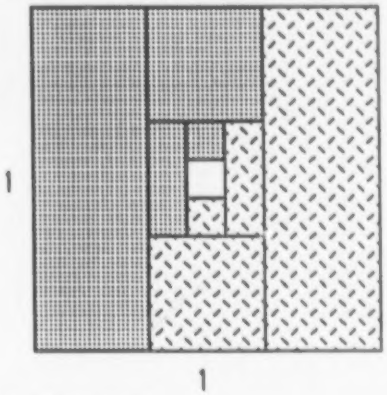


Other geometric series can also be explored.

$$S_n = \sum_{k=0}^{n} 1/a^k \qquad (a>1)$$

Take for example a = 3
$S_1 = 1/3$
$S_2 = 1/3 + 1/9$
$S_3 = 1/3 + 1/9 + 1/27$
. . .

There is also a geometric interpretation for this series. What is the limit?



Activity 12

Explore the following series:

$$S_n = \sum_{k=0}^{n} 1/k!$$

Casio Fx 7000G

$1\ \boxed{\div}\ 0\ \boxed{\text{SHIFT}}\ \boxed{x!}\ \boxed{\text{EXE}}\quad \boxed{+}\ 1\ \boxed{\div}\ 1\ \boxed{\text{SHIFT}}\ \boxed{x!}\ \boxed{\text{EXE}}\ \ldots$

This sequence approaches its limit (the number e) very quickly. Here are the first 12 terms of the sequence

$S_1 = 2$
$S_2 = 2.5$
$S_3 = 2.666666666$
$S_4 = 2.708333333$
$S_5 = 2.716666666$
$S_6 = 2.718055555$
$S_7 = 2.718253968$
$S_8 = 2.718278769$
$S_9 = 2.718281525$
$S_{10} = 2.718281801$
$S_{11} = 2.718281826$
$S_{12} = 2.718281828$

Activity 13

$S_1 = 1$
$S_2 = 1 + 1/2$
$S_3 = 1 + 1/2 + 1/3$
. . .
$S_n = 1 + 1/2 + 1/3 + 1/4 + \ldots + 1/n$

That is,

$$S_n = \sum_{k=1}^{n} 1/k$$

Sharp EL-506S

$1\ \boxed{1/x}\ \boxed{M+}\ \boxed{RM}\quad 2\ \boxed{1/x}\ \boxed{M+}\ \boxed{RM}\quad 3\ \boxed{1/x}\ \boxed{M+}\ \boxed{RM}\ \ldots$

Casio Fx 7000G

$1\ \boxed{+}\ 1\ \boxed{\div}\ 2\ \boxed{EXE}\quad \boxed{+}\ 1\ \boxed{\div}\ 3\ \boxed{EXE}\quad \boxed{+}\ 1\ \boxed{\div}\ 4\ \boxed{EXE}\ \ldots$

$S_1 = 1$
$S_2 = 1.5$
$S_3 = 1.83\ldots$
$S_4 = 2.08\ldots$
$S_5 = 2.28\ldots$
$S_6 = 2.45$
$S_7 = 2.59\ldots$
$S_8 = 2.71\ldots$
$S_9 = 2.82\ldots$
$S_{10} = 2.92\ldots$
$S_{11} = 3.01\ldots$
$S_{12} = 3.10\ldots$

This is an increasing sequence. To see that it grows without bounds, we can compare it with another sequence

$$\begin{aligned} & 1 + 1/2 + 1/3 + 1/4 + 1/5 + 1/6 + 1/7 + 1/8 + \ldots. \\ > \ & 1 + 1/2 + 1/4 + 1/4 + 1/8 + 1/8 + 1/8 + 1/8 + \ldots. \\ = \ & 1 + 1/2 \quad + \quad 1/2 \qquad\qquad + 1/2 \qquad\quad + \end{aligned}$$

Part 4

Explore the following sequences. Get the first few terms and also use large values of *n* to try to guess their limits.

Activity 14

For the next sequence, be sure your calculator is set to use radians (instead of degrees) for the angles.

$S_n = \sin(1/n) \times n$

The key strokes for the second term are:

Casio Fx 7000G

$\boxed{SIN}\ \boxed{(}\ 1\ \boxed{\div}\ 2\ \boxed{)}\ \boxed{\times}\ 2\ \boxed{EXE}$

Use the left arrow key to retrieve the last expression. $\boxed{\Leftarrow}$

sin (1÷2) x 2

Use the left arrow again to place the cursor at the appropriate place and overwrite; you can also insert using the SHIFT and the INSERT keys.

Casio Fx 7000G

$\boxed{\Leftarrow}\qquad \boxed{SHIFT}\ \boxed{INS}$

Write down the terms

$S_1 = 0.841470984$
$S_2 = 0.958851077$
$S_3 = 0.981584090$

$S_4 = 0.989615837$
$S_5 = 0.993346653$
$S_6 = 0.995376796$
$S_7 = 0.996602108$
$S_8 = 0.997397867$
$S_9 = 0.997943656$
$S_{10} = 0.998334166$ (note that the difference from 1 is less than $1/10^2$)
$S_{100} = 0.999983333$ (the difference from 1 is less than $1/100^2$)
$S_{1000} = 0.999999833$ (the difference from 1 is less than $1/1000^2$)

Activity 15
Obtain some terms of the following sequence:
$S_n = (1 + 1/n)^n$
Casio Fx 7000G

$\boxed{(}\ 1\ \boxed{+}\ 1\ \boxed{\div}\ 12\ \boxed{)}\quad \boxed{x^y}\ 12\ \boxed{\text{EXE}}$

$\boxed{\Leftarrow}$

Use the left arrow to retrieve the last expression.
$(1 + 1 + 2)\ x^y\ 2$
Use the left arrow again and the SHIFT and the INSERT keys to obtain other terms of the sequence.

This sequence has the same limit as the sequence in Activity 12. However, this sequence approaches the limit much more slowly; that is, the corresponding terms of the sequence in Activity 12 are much closer to the limit.
$S_{12} = (1 + 1/12)^{12} = 2.61303529$
(The 12th term gives us only one place correct; whereas, in the other sequence all the places displayed by the calculator were correct.)
. . .
$S_{100} = (1 + 1/100)^{100} = 2.70481383$
(After a hundred terms we have only two figures correct.)

Activity 16
$S_n = k^{1/n}$ $(0 < k)$
Take, for example, $k = 5$. For the third term, for instance, push the keys:
Casio Fx 7000G

$3\ \boxed{\sqrt[x]{\ }}\ 5\ \boxed{\text{EXE}}$

Write down the successive terms.
$S_1 = 5$
$S_2 = 2.23\ldots$
$S_3 = 1.70\ldots$
$S_4 = 1.49\ldots$
$S_5 = 1.37\ldots$
. . .
$S_{10} = 1.17\ldots$
$S_{100} = 1.01\ldots$

Activity 17

$S_n = n^{1/n}$

To obtain the 10th term, for example, you can push the keys:
Casio Fx 7000G

10 $\boxed{\sqrt[x]{\ }}$ 10 $\boxed{\text{EXE}}$

$S_1 = 1$
$S_2 = 1.414\ldots$
$S_3 = 1.442\ldots$
$S_4 = 1.414\ldots$
$S_5 = 1.379\ldots$
$S_6 = 1.348\ldots$
. . .
$S_{10} = 1.258\ldots$
$S_{100} = 1.047\ldots$
$S_{1000} = 1.0069\ldots$

Part 5: Zooming In: Limits of Functions

Part 6

For each of the sequences in Activities 1 - 17
a) Find the limit L (or determine that it does not exist).
b) If the limit L exists, find a number N such that if $n > N$, then $| S_n - L | < 0.001$.
c) If the limit L exists, give a number N, such that, if $n > N$, then $| S_n - L | < 0.000001171717$.

APPENDIX A.2

LINEAR FUNCTIONS

Linear Functions
Math 121

Linear functions are often useful for representing simple problems. For a definition of the term *function* and examples of many different kinds of functions, see pp. 5-20 of the text.

Use your graphing calculator to graph the functions in the following problems. It is also a good idea to keep track of your graphs with pencil and paper.

Activity 1

A car rental company charges $15 plus $0.20 a mile to rent a van for a day. Show the relationship between the miles driven and the cost by using a table, a graph, and an equation. Explain to someone else how to use the function you have described to determine:

the cost if you are given the miles driven;
the miles driven if you are given the total cost.

If the graph does not appear in your calculator's window, zoom out (see instructions for calculator) until it appears, or reset your range to [-150, 150] and [-100, 100].

Describe the significance of the numbers in the problem for the properties of the graph.

This company also rents trucks for $40 plus $0.40 a mile. Describe the cost of the truck in terms of miles driven as you did before.

Graph this function on the same screen as the first, and compare the two graphs.

If you can do a job just as well with three vans as with one truck, which is cheaper? Construct the functions that are involved, and justify your answer in more than one way. Use the graphing calculator for one justification. If you are ready to learn about the many features of your calculator, use the trace and automatic zoom features to solve this problem.

Activity 2

Clear the screen of your graphing calculator, reset the range if needed, and graph functions that describe the following situations:

How far Big Bear travels at 4 miles per hour.
How far Medium Bear travels at 2 miles per hour.
How far Baby Bear travels at 1 mile per hour.
How far Old Bear travels at 0.5 miles per hour.
How far Snail travels at 0.2 miles per hour.

Interpret the differences in the "slope" of the lines. How far does each traveler go in 5 hours? How long does it take each to go 4 miles?

Activity 3

The host provided 3 gallons of punch at a party, and people drank it at the rate of 12 gallons per hour. Express the amount of punch remaining πas function of time, and graph the function.

If people drank at the rate of 0.5 gallons an hour, what would the graph of the function look like?

Discuss the "slope" of these two graphs, and think about the differences between these functions and those in Activities 1 and 2.

Activity 4

A T-shirt company has fixed costs (rent, etc.) of $200,000 per year. The cost of producing one T-shirt is $1.50. Find the total annual cost of producing any number of T-shirts and graph the function.

They sell the T-shirts for $4 each. Find a function that gives the total revenue and graph it. When does the company break even?

If you use the calculator to graph these functions, does it help?

Activity 5

In the previous activities, for what values does each function make sense?

For further discussion of the ideas from Activities 1-5, study pp. 43-51 in the text.

APPENDIX A.3

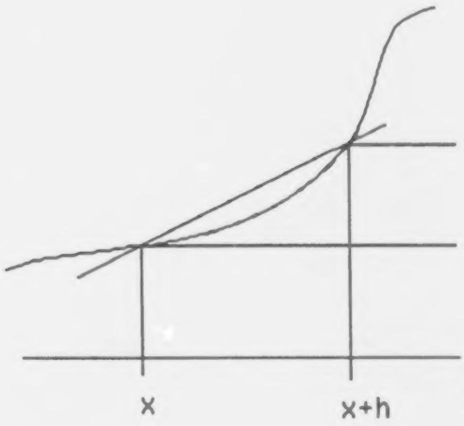
EVALUATION INSTRUMENTS

Quiz 1 (Suggested questions for quiz)

1) On the attached figure, the curved line represents the graph of a function
$y = f(x)$.

Give the geometric representation of the following
h
$f(x)$
$f(x+h)$
$f(x+h) - f(x)$

$$\frac{f(x+h) - f(x)}{h}$$



2) If x represents time, and $y = f(x)$ is a function representing distance travelled, what do the following represent
h
$f(x)$
$f(x+h)$
$f(x+h) - f(x)$

$$\frac{f(x+h) - f(x)}{h}$$

3) For the following sequences:
i) Find their limit L.
ii) Use your calculator (or any other method) to find a value of n such that the distance from S_n to the limit is less than 0.001
iii) less than 0.000001
a) $S_n = (2^n - 1)/2^n$
b) $S_n = 1/n!$
c) $S_n = (-1)^n / n$
d) $S_1 = 1$, $S_n = 1 + 1/S_{n-1}$

Final Quiz: Calculus 121

1) Find the area under the given curve between the given values.

$$\int_{0}^{2} x^3 \, dx$$

$$\int_{1}^{3} 1/x \, dx$$

$$\int_{0}^{\pi} \sin x \, dx$$

2) Let $y = f(x)$ and $\Delta x = (b - a)/n$

Describe what the following represent geometrically on the diagram:

a) $f(x_i) \Delta x$

b) $\mathop{S}\limits_{i=1}^{n} f(x_i) \Delta x$

c) $\lim\limits_{n \to \infty} \mathop{S}\limits_{i=1}^{n} f(x_i) \Delta x$

3) Let $A(x)$ be the area under the curve $y = f(x)$ between the fixed value **a** and the variable value **x**

$$A(x) = \int_{a}^{x} f(u) \, du$$

Represent the following clearly on a diagram

$A(x)$

$A(x + h)$

$A(x + h) - A(x)$

b) What does $\dfrac{A(x + h) - A(x)}{h}$ represent?

c) To what value does $\dfrac{A(x + h) \quad A(x)}{h}$ approach as $h \to 0$?

4) Describe in your own words the relationship between the integral and the derivative (fundamental theorem of calculus).

Final Exam: Calculus 121

Note: Some questions from the final exam were given to both the experimental and control students.

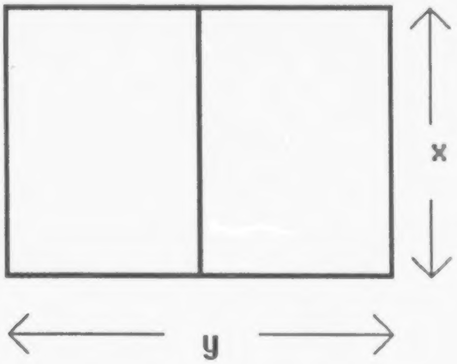
1) Let $y = x^3 - 3x^2 + 2$

a) Sketch the graph of the function, indicating maxima and minima.

b) Find the intervals where the function is increasing

2) Find the minimum amount of fencing necessary to construct two adjacent rectangular pens if the area to be enclosed is 2400 feet.

Hint:



3) A helicopter rises straight up in the air, and after t seconds its distance from the ground in feet is given by

$$s(t) = 14\sqrt{t} - t$$

a) How long will it take the helicopter to rise to a height of 45 feet?

b) Find the velocity when the helicopter has risen to a height of 45 feet.

c) Find the maximum height reached by the helicopter.

4) Find the derivatives of the following functions

a) $y = (3x + 2)^2$

b) $y = x * \sin(x)$

c) $y = 1/x$

5) Sketch the following integrals as the area under the graph of the given function, between the given numbers, and calculate their values

$$\int_1^2 x^2\,dx$$

$$\int_1^4 1/x\,dx$$

$$\int_0^{\pi/2} \cos x\,dx$$

6) Choose **two** of the following concepts:

i) derivative
ii) integral
iii) limit

a) How would you present the first concept to a student?
(You may use your own words; draw diagrams; use convenient notation; use particular cases of the concept; give specific examples; give geometrical, physical, or other concrete interpretations of the concept; etc.)

b) How would you present the second concept to a student?

c) Explain the relationship between the two concepts you chose. (You may use your own words, but be as explicit as possible.)

APPENDIX B.1

Using Principles from Cognitive Psychology to Guide Rational Number Instruction for Prospective Teachers

Judith T. Sowder, Nadine Bezuk, and Larry K. Sowder
Center for Research In Mathematics and Science Education
San Diego State University

The preparation of this manuscript was partly supported by National Science Foundation (Grant No. TPE 8950315). The opinions expressed here do not necessarily reflect the position, policy, or endorsement of the National Science Foundation.

Abstract

Mathematics content courses for elementary teachers need to be revised to prepare teachers to implement the reforms being currently recommended. The revision of such a course is described. The traditional orientation to rational numbers in courses for prospective teachers focuses on definitions and rules for operations. A review of the recent content analyses of rational numbers and of research on learning of rational numbers indicated needed changes in the curriculum on rational numbers in a course for prospective teachers. Principles from cognitive psychology, useful in determining ways to motivate prospective teachers and assist them in extending their procedural knowledge of rational numbers to knowledge that is adaptive and reflective of rational number sense, guided planning for revising instruction.

Mathematics content courses for prospective elementary teachers are traditionally taught in many universities as lecture courses, often to classes of over 100 students. Manipulative materials are rarely available due to class size, the instructors' lack of familiarity with manipulatives, and/or the belief that using manipulative materials with prospective teachers is appropriate only in a methods course. The coverage of rational numbers is fairly uniform in the textbooks used in these courses: A chapter on fractions defines fractions as ordered pairs, discrete and continuous models are presented, definitions are given for fraction equality and for algorithms for operations, properties are stated, and end-of-the-chapter exercises provide some opportunities to explore the concepts introduced in the chapter and to apply the definitions to comparing and operating on fractions. A similar chapter on decimals follows the one on fractions, and finally, rational numbers are defined and field and order properties are formally stated.

The call for implementation of new curriculum and teaching standards for school mathematics (National Council of Teachers of Mathematics, 1989a, 1989b) carries both an explicit and an implicit charge to reorganize our teacher preparation programs. It is increasingly clear that course content needs to go beyond rules and definitions to allow prospective teachers to explore the interconnections that exist in mathematics, to reason about mathematics, to communicate mathematically, and to come to a better understanding of the nature of mathematics. It is doubtful that current instruction on rational numbers, as described above, will lead prospective teachers to be able to reason about rational numbers or to understand the role of rational numbers within the mathematics of quantity. If we accept the statement found in *Everybody Counts* (National Research Council, 1989) that the major objective of elementary school mathematics is to develop number sense, then we must provide prospective teachers the opportunities to develop their own number sense. This is particularly true for rational number sense, which seems less likely than whole number sense to develop without instructional intervention.

Likewise, the *manner* in which mathematics is taught to prospective teachers needs to be reexamined. "Experiences mathematics teachers have while learning mathematics have a powerful impact on the education they provide in the classroom....Those from whom they are learning are role models who contribute to a growing vision of what it means to teach mathematics successfully--or unsuccessfully (NCTM, 1989b, p. 66).

In this chapter, we present an account of our planning for part of a content course for prospective elementary teachers, with a content focus on rational numbers. We were particularly interested in examining current research in cognitive psychology to see if there were principles from that research area that were appropriate for our course planning. We of course also examined the research on rational number learning. The following sections review the relevant literature in these areas and indicate how we applied the research to our course planning. The first section, on rethinking teacher preparation, is intended to be introductory and to provide information about our understanding of the particular challenges of a course focusing on rational number. The second section reviews the mathematical interpretations of rational number and research on rational number learning. In the third section, we share cognitive principles derived from three sources that we found to be particularly useful in our planning. We summarize the subsections with statements about how we perceive the research and cognitive principles to be applicable to our course planning. We close with a discussion of the alignment of the principles and the applications presented here with current recommendations regarding the place of rational numbers in the school curriculum.

It was our decision not to include a detailed course description. The applications stated here could be used in a variety of ways to structure the work with rational numbers, depending upon the local interpretations of the applications and the local constraints such as class size and available resources. Our plan can not be offered as a model of *the* best way to structure such a course.

Rethinking Teacher Preparation

In recent years, cognitive theory and research have contributed to our understanding of the learning process. The advances made in formulating and testing models of the kinds of information structures students acquire when learning mathematics have made it possible for us to gain insights into the development of mathematical ideas (Sowder et al., 1989). Linn (1986) has pointed out that science (including mathematics) education would be strengthened by building upon what we already know about the cognitive structure of the subject matter. The work of Carpenter and colleagues (e.g., Carpenter, Fennema, Peterson, Chiang, & Loef, 1989) provides an example of how cognitive learning research can be incorporated into instructional planning. They have shown that when teachers possess detailed knowledge about children's thinking and problem solving, their knowledge of their students' thinking processes and their planning for instruction are profoundly affected.

Shulman (1987), in his work on teacher knowledge, has pointed out that such research-based knowledge is at the very heart of his definition of pedagogical content knowledge, a blend of content and pedagogy that demands an understanding of the content in terms of how it can be organized and represented for instruction. Pedagogical content knowledge "includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning" (Shulman, 1986, p. 9).

One avenue for teachers to acquire this knowledge is to become aware of their own thinking and problem solving processes. Yet rarely are students in mathematics courses required to examine their own learning processes. Norman (1980), in reflecting upon expectations of university students in general, noted that: ."We expect students to learn yet seldom teach them about learning. We expect students to solve problems yet seldom teach them about problem solving. And, similarly, we sometimes require students to remember a considerable body of material yet seldom teach them the art of memory" (p. 97). In courses designed for prospective teachers, a focus on awareness of one's own learning processes is particularly critical. Teachers should be able to reflect on their own knowledge and learning strategies when making decisions about classroom teaching (Peterson, 1988). Their understanding of how they learn mathematics should affect the way in which they teach mathematics.

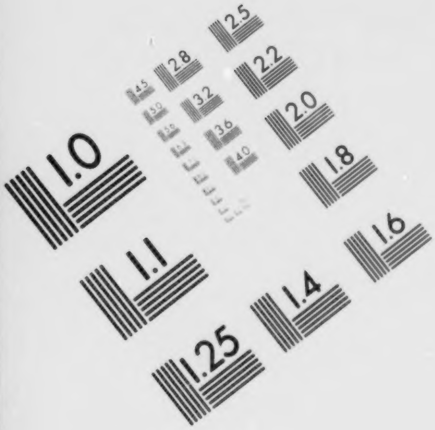
One frequent theme of current psychological research is to view the learner as the active constructor of his or her own knowledge. This theme was recently acknowledged in *Everybody Counts:* "Clear presentations by themselves are inadequate to replace existing misconceptions with correct ideas. What students have constructed for themselves, however inadequate it may be, is often too deeply ingrained to be dislodged with a lecture followed by a few exercises" (National Research Council, 1989, p. 60). A number of studies document how individuals organize the knowledge they acquire and how that organization changes with time and experience; summaries are provided, for example, by Resnick (1986) and by Cobb (1987). A particularly interesting aspect of this research is the study of student preconceptions and misconceptions in mathematics (Davis, 1984; Novak, 1987; Shaughnessy, 1985). Not until teachers are aware of the student conceptions that interfere with a full understanding of fundamental ideas in mathematics and science can they go about creating the instructional conditions that allow for change and transformation of those conceptions.

Yet those of us who prepare and work with elementary teachers know that they themselves sometimes harbor these same misconceptions. For example, the work of Post and his colleagues (Post, Harel, Behr, & Lesh, 1988) has shown that many intermediate-level teachers have difficulty with conceptual and computational rational number questions, and are unable to explain their solution processes in more than a procedural manner. The errors these teachers make are probably not due to "unsureness, carelessness, or unique situational conditions [but rather] are the result or the product of previous experience in the mathematics classroom " (Radatz, p. 16, 1980). Teachers whose mathematics preparation did not allow them the opportunity to explore and reflect upon rational number concepts they formulated during their own childhoods can hardly be faulted for continuing to carry misconceptions and the resulting error patterns with them into their classrooms. Systematic errors are not easy to eradicate (Resnick & Omanson, 1987), particularly without assistance. The pervasive and resilient nature of misconceptions is well established (Confrey, 1987).

In our experience, many prospective teachers, and many mathematics faculty, think of the required university-level courses on mathematics for elementary teachers as opportunities to review concepts and brush up on skills. But when such a course moves beyond review of content to allow prospective teachers to examine their own understanding of the content and to explore and redefine the content in a manner that will allow them to teach that content, it can perhaps be one of the most challenging courses to teach within a department of mathematics. That some prospective teachers believe they already *know* the content because they are procedurally competent, while others believe that they are incapable of understanding the content, only increases the challenge. Many hold conceptions about the nature of mathematics, its structure, and what comprehending mathematics means that are at variance with those of the mathematics community.

Entering prospective teachers have acquired extensive procedural knowledge about rational numbers. As our data on entering students shows, they have learned both correct and incorrect rules for finding equivalent fractions, for performing operations on fractions and decimal numbers, for comparing and ordering rational numbers. They are most familiar with the part-whole construct of rational numbers (Silver, 1981), at least for fractions between 0 and 1, represented as the shaded portion of a geometric region, preferably a circle. The most adept have what Hatano (1988) refers to as *routine expertise*, that is, they are able to carry out procedures quickly and accurately. While routine expertise possessed by prospective teachers might be sufficient for much of their own day-to-day work with rational numbers, it does not allow them to go beyond teaching for procedural understanding. In contrast, we would hope that a teacher has some degree of *adaptive expertise*, which Hatano defines as understanding how and why given procedures works, how they can be adapted to new situations, and when they can be abandoned in favor of another procedure. This type of expertise allows the content to be understood in terms of how it should be organized for instruction, that is, it allows for the development of pedagogical content knowledge.

Application: Prospective teachers need to be provided with opportunities to examine their personal understandings of rational number concepts. This should be done in the context of problem situations that force them to confront any misconceptions they carry from earlier


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0

AIIM

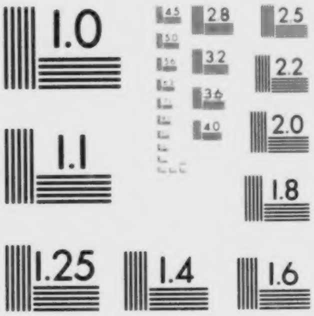
Association for Information and Image Management

1100 Wayne Avenue, Suite 1100
Silver Spring, Maryland 20910

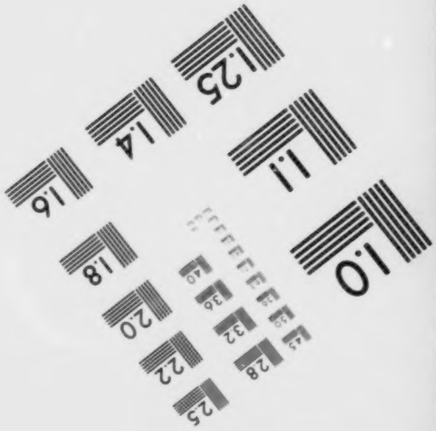
301/587-8202

1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0

Centimeter
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
mm
1
2
3
4
5
Inches


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5
5.0

1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5


1.0
1.1
1.25
1.4
1.6
1.8
2.0
2.2
2.5
2.8
3.2
3.6
4.0
4.5

MANUFACTURED TO AIIM STANDARDS
BY APPLIED IMAGE, INC.

experiences, to come to new understandings of connections and relationships that underlie the mathematics of rational numbers, and to reflect upon these new understandings and how they were reached.

Content Analysis of Rational Numbers

In their analysis of the major themes of the Research Agenda Project's conference on middle school number concepts and operations, Hiebert and Behr (1988) noted that existing research "does not prescribe instruction in an exact way, but it does set parameters within which effective instruction is likely to fall" (p. 12). They made three pertinent observations: instruction is often too limited and symbol-oriented (rather than meaning-oriented); instruction should not deliver knowledge in prepackaged form, but instead students should construct their own knowledge; and instruction should provide students with structured learning experiences in order to acquire essential conceptual and procedural knowledge. They recommended that instruction devote increased attention to developing the meaning of symbolic notations such as fraction symbols, developing concepts such as order and equivalence that are important in fostering a sense of the relative size of fractions, helping learners connect their intuitive understandings and strategies to more general, formal methods, and promoting "the development of powerful solution strategies to solve complex problems" (p. 13). They commented that instructional activities that support the process of linking symbols with concrete materials, including meaningful actions on these materials, and verbal interaction such as talking about the mathematics students are doing, seem to be especially successful.

In the same volume, Kieren (1988) suggested that a teacher's role is to expose children to the "critical aspects of the rational number world" (p. 177). The teacher must carefully vary the aspects of rational numbers that are dealt with in instruction, making sure that instruction is not limited to one interpretation of fractions. He also recommended that instruction in operations on fractions be built on children's intuitive understanding of fractions and based upon actions on objects, rather than being based solely on the manipulation of symbols according to a set of rules and procedures. He noted that premature formalism leads to symbolic knowledge which children cannot connect to the real world, resulting in the virtual elimination of any possibility for children to develop number sense about fractions and operations on fractions. He pointed out that symbolic knowledge that is not based on understanding is "highly dependent on memory and subject to deterioration" (p. 178).

The above comments appear to be equally applicable to courses for prospective teachers. The recommendations would be difficult to carry out in elementary classrooms if the teachers have not experienced instruction that allowed them to develop their own meanings through opportunities to connect intuitive understanding to more formal methods. Prospective teachers who have experienced instruction where premature formalism prevented them from developing rational number sense need to be provided with different experiences that will allow them to link symbols with meaningful actions on concrete materials and to share with classmates the understandings that evolve from these actions.

In order to devise experiences that would provide the needed learning opportunities we reviewed the research on rational number learning. Most of this research has focused on understanding of the different subconstructs of rational number, the effect of different representations on learning rational number, number size concepts--that is, comparing and ordering rational numbers, equivalence of rational numbers, estimating with rational numbers, and operations on rational numbers. Only that research we found to be relevant to teacher preparation is discussed here.

Various Interpretations of Rational Number

Kieren (1976) identified six basic ways in which rational numbers can be interpreted, which he termed "subconstructs": as a measure, a decimal, a quotient or indicated division, a ratio, an operator, and a part-to-whole comparison. These six subconstructs have similar mathematical properties, but they have been shown to elicit different types of responses from students. Kieren recommended that all six subconstructs be present in any well-designed mathematics curriculum.

He insisted that true understanding of fractions requires both an understanding of each of these subconstructs and of their interrelationships. He also recommended that knowledge of rational numbers be built up from experiences with a variety of basic ideas of rational numbers and that curricula be designed to include all six subconstructs of rational number.

In addition to the consideration of different subconstructs of rational number, we can think of different ways of representing rational numbers, and the models usually used within these representations. Lesh, Landau, and Hamilton (1980) discussed five modes of representation: real-world objects, manipulative materials, pictures, spoken symbols, and written symbols. For example, the number one-third can be represented in the real-world-object mode by showing a pizza and asking what amount each of three people would get if they shared it equally; in the manipulative-materials mode with pattern blocks by showing that three blue diamonds cover the same area as one yellow (unit) hexagon, then removing two of the blue diamonds; in the pictorial mode by drawing a picture of a rectangle with one-third of the area shaded, or showing the point one-third on the number line; in the spoken-symbol mode by saying "one-third"; and in the written-symbol mode by writing "one-third" or "1/3." Lesh and his colleagues suggested that the process of making translations between and within modes of representation enhances students' flexibility of thought regarding the concepts being studied.

In order to use these modes of representation effectively, students need to understand the models we use to represent rational numbers. The models of the fraction concept that students are usually exposed to in elementary school appear not to be of sufficient variety to encourage generalization of the concept. Novillis (1976) found, for example, that many students were able to associate correctly the fraction 1/5 with a set of five objects, one of which was shaded, but most students were not able to associate the fraction 1/5 with a set of ten objects, two of which were shaded, even when the objects were arranged so that it was fairly easy to see that one out of every five objects was shaded. In a later study (Novillis-Larson, 1980), she found that students seem to confuse the number line model with the part-whole model. These difficulties indicate a lack of flexibility in students' understanding of fractions.

There is some evidence that the part-whole model of rational number dominates the thinking of most individuals, including adults. Silver (1981) found that young adults' understanding of fractions was limited to one model, that of parts of a circle. Similarly, Kerslake (1986) found that the part-whole model was the only one with which all of the 13- and 14-year-olds surveyed were familiar. These students also were unfamiliar with and did not readily accept the quotient subconstruct of fractions, that is, that three-fourths can mean three divided by four, and many were unable to attach any meaning to equivalent fractions. They found it difficult to believe that a fraction is a number and often said that fractions were just "two numbers put on top of one another" (p. 91). Kerslake reported receiving similar responses from teachers.

Application: Instruction for prospective teachers should include experiences with all major subconstructs of rational number. A rich variety of models and representations of rational numbers should be used. In particular, prospective teachers should be made to understand that the part-whole model is not the only useful interpretation for fractions, and that in fact other models are more appropriate for demonstrating certain concepts and operations.

Number Size: Comparing, Ordering, and Estimating

The importance of the development of a quantitative notion, or an awareness of the "bigness", for fractions and for decimals has been noted in several research reports (Behr, Wachsmuth, & Post, 1985; Sowder & Markovits, 1989). Sowder (in press) has suggested that the ownership of this notion is the essence of rational number sense. Measures of this understanding in the realm of fractions include children's ability to perceive the relative size of fractions, to compare and order fractions using size concepts and benchmarks, to find or recognize equivalent fractions, and to estimate the location of a fraction on a number line.

The Rational Number Project investigators have devoted a considerable amount of attention to the development of children's understanding of ordering fractions (for example, see Behr, Wachsmuth, Post, & Lesh, 1984). Of particular interest are the four types of reasoning processes children used in completing comparison tasks: (a) considering both the numerator and denominator of each fraction, (b) referring to manipulative aids, (c) using a reference point (such as 1/2) to compare two fractions, and (d) improperly using one's knowledge of whole numbers, which is referred to as whole number dominance. They recommended that more time be devoted to developing the meaning of fractions, focusing first on unit fractions (i.e., fractions whose numerator is one), then proceeding to ordering unit fractions, and finally to non-unit fractions. Recent research by Post, Harel, Behr, and Lesh (1988) indicates that intermediate-level teachers have difficulty ordering sets of fractions. This is not surprising, in light of the little attention traditionally given to this topic in teacher preparation courses. When the topic is attended to in current texts, it is frequently in terms of cross-multiplication rules for comparing two fractions, rather than in terms of benchmarks and application of number sense.

Attention has also been given to comparing and ordering decimal numbers. Sackur-Grisvard and Leonard (1985) identified two common types of errors students make in comparing decimal numbers with the same whole number parts: (a) comparing the decimal portions as if they were whole numbers (e.g., 12.4 < 12.17, since 4 < 17), (b) choosing the number with more decimal places as the smaller (e.g., 12.94 < 12.7, since 12.94 has hundredths, while 12.7 has only tenths). They state that these rules are stable intermediate organizations of knowledge developed as children are in the process of learning the concept of ordering decimals. They believe that these structures are persistent, often in use for at least three to four years. The authors encouraged teachers to examine the exercises used in instruction, making sure that students cannot successfully complete them by using these rules alone.

Quantitative understanding is important in evaluating the reasonableness of results of computation involving fractions. For example, many students will incorrectly report that 1/2 + 1/3 = 2/5, by adding the numerators and the denominators. But students who understand something about the "bigness" of fractions will realize that 2/5 is not a reasonable answer; the problem involved adding something to 1/2, so the answer should be greater than 1/2, but 2/5 is less than 1/2. The development of this quantitative notion of fractions should be a priority of instruction on fractions.

Application: In a course for prospective teachers, considerably more class time than in the past should be spent (a) on developing number size concepts for rational numbers, (b) on ordering rational numbers, and (c) on estimating answers to operations on rational numbers. The development of rational number sense should be a major objective of the course.

Operations on Rational Numbers

Most research on the teaching and learning of fractions has focused on concepts, order, and equivalence, rather than operations. The research on students' understanding of fractions and their ability to perform operations on fractions has shown disappointingly poor results. For example, based on interviews with 20 6th-grade students, Peck and Jencks (1981) reported that children could follow rules for operating on fractions, but did not understand why the rules worked. They recommended that the emphasis in instruction shift from learning the rules for operations to understanding fraction concepts.

Likewise, Silver (1981) found that the algorithms used by community college students for comparing and adding fractions were not connected to the representations they used in interpreting fractions. Most of the community college students in his study were unable to explain procedures they used for fraction computation. He recommended that instruction stress the connections between the models students have learned and the process of comparing and operating on fractions.

Common misunderstandings related to multiplication and division are that "multiplication makes bigger," meaning that students expect the product to be greater than both factors, as in most whole number multiplications, and "division makes smaller," meaning that students expect the quotient to be less than the dividend, as in most whole number division (e.g., Greer, 1988; Sowder, 1988). For example, students often will report the answer to 6 ÷ 1/2 as 3. Many students think that 12 is not a reasonable answer because they believe that division makes smaller and therefore the answer should be less than 6. This error is also prevalent in the thinking of preservice elementary teachers (Graeber & Tirosh, 1988).

Decimal computation is also a source of difficulty in school mathematics. Hiebert and Wearne (1985) report that students' decimal computation procedures are frequently syntactically rather than semantically based. It seems that "students' improved performance with age results from an increasing facility with the syntax rather than from a better conceptual understanding" (p. 200). In our own classes for prospective teachers, we frequently find that decimal-computation decisions are more frequently guided by rules they have memorized than by sense-making. For example, when asked to place the decimal point in a product of two decimal numbers where additional zeros have been placed, they will usually apply the rule for counting decimal places and give an answer which is obviously unreasonable.

Application: Prospective teachers need to learn how to make a correct choice of operations, particularly in the cases where the "multiplication makes bigger, division makes smaller" error might be made. This common error can be used to introduce and overcome the cognitive conflict associated with the error. Instruction on algorithms should be semantically rather than syntactically based.

Cognitive Science Principles Applicable to Structuring a Course for Teachers

The two major goals for our course, that is, that prospective teachers should acquire an extensive, interrelated knowledge of the domain of rational numbers, and that they should develop an understanding of how they acquired this knowledge, guided the selection of cognitive principles relevant to our planning. Three sources in particular offered us assistance in formulating instructional guidelines that fit within a cognitive science framework and that were based on cognitive research. The applications we took from these sources are next discussed in some detail.

Intentional Learning

In the first instance (Scardamalia, Bereiter, McLean, Swallow, & Woodruff, 1989), a set of principles from cognitive research was developed to guide the design of computer environments.

The research underlying these principles dealt with learning in a more general setting, however, so that the principles are applicable to the design of most instruction, whether computer oriented or not. Using contributions to the Chipman, Segal, and Glaser (1985) volume, Scardamalia et al. found that successful learners "use a variety of cognitive strategies and self-management procedures to pursue knowledge-related goals, to relate new knowledge to old, to monitor their understanding, to infer unstated information, and to review, reorganize, and reconsider their knowledge" (p. 53). In contrast, the approach to learning by those who are less successful is characterized as "an additive rather than a transformational process" (p. 53), as focusing on surface features, and as organized around topics rather than around goals. The authors note the persistence of immature strategies which have led students to partial success, even though they are inadequate for all situations. As discussed earlier, such strategies are common among many prospective elementary teachers who have had some success with memorizing procedures for performing rational numbers operations.

Building on experiments that led from immature to active and successful learning strategies, Scardamalia et al. advocated a teaching approach that began with modeling and explanation of strategies, together with encouragement of students to take greater responsibility for their own learning. In particular, they advocated a theory-based instructional approach they called procedural facilitation which provided support while learners tried to adopt more complex learning strategies. "These supports include turning normally covert processes into overt processes; reducing potentially infinite sets of choices to limited, developmentally appropriate sets; providing aids to memory; and structuring procedures so as to make it easier to escape from habitual patterns" (p. 54). The supports were designed to lead eventually to independent processing by learners.

This teaching approach was summarized by Scardamalia et al. within a set of instructional design principles. The ten principles relevant to planning our course are listed here together with the manner in which they apply to the design of instruction on rational numbers.

1. *Make knowledge-construction activities overt.*

Application: Students should notice when they are connecting old and new knowledge of rational numbers, when they are solving problems based on understanding rather than on procedural knowledge, and how they go about setting goals for themselves related to acquiring understanding.

2. *Maintain attention to cognitive goals.*

Application: Students should be asked to formulate their personal learning goals, particularly during the time allocated to reflection on a lesson. They should be asked to sort through what they did and did not understand, and state what they need to do or know in order to acquire understanding.

3. *Treat knowledge lacks in a positive way.*

Application: A lack of understanding should not be treated as a failure. Students need to be assisted in examining what they know and what they do not know. Too often students with some degree of procedural facility do not realize when they do not understand a particular concept and how it relates to other concepts; as a result they typically characterize errors as "dumb" or "stupid."

4. *Provide process-relevant feedback.*

Application: This is a difficult goal, but to some extent it can be met through group work in which students question one another and examine each other's learning processes.

5. *Encourage learning strategies other than rehearsal.*

Application: Learning strategies should be aimed at understanding rather than memorizing rules for operating on rational numbers. Paraphrasing, applying knowledge to novel problems, explaining to peers, creating assessment items, and making predictions are all alternatives worth exploring.

6. *Encourage multiple passes through information.*

Application: Naive learners typically do not return to completed problems. Students should be expected to examine one another's problem-solving processes on particular rational number problems, and write reflectively on problems they themselves have solved.

7. *Support varied ways for students to organize knowledge.*

Application: Students need to be assisted and encouraged to connect new learning to prior knowledge, to examine relationships, and to reflect on their learning. They should be forming a richly connected schema, rather than unrelated individual concepts.

8. *Encourage maximum use and examination of existing knowledge.*

Application: If existing knowledge of rational numbers is not examined, new knowledge will be independent of it. The old knowledge, including its misconceptions, will continue to be called up into working memory unless it is examined, extended and expanded so that correct conceptions will be stronger and more likely to be retrieved.

9. *Provide opportunities for reflectivity and individual learning styles.*

Application: Individual students need some control over how class activities and assignments are structured. Some need more "thinking time" that others. In addition, they need to reflect on how the activities promote or slow their understanding. Students should be asked to reflect on class discussions, group solutions to problems, and their own understanding.

10. *Give students more responsibility for contributing to each other's learning.*

Application: The class structure should emphasize cooperative learning. As much as possible, students should be made to feel responsible for their own learning and for the learning of others in their group.

Enhancing Motivation for Comprehension

Working within the framework of cognitive instructional psychology, Hatano and Inagaki (1987) have constructed a model for motivating comprehension, based on the earlier theory of Berlyne (1963, 1965). According to this model, an individual in a state of cognitive incongruity is aware that comprehension is inadequate but is within reach, and becomes motivated to seek satisfactory explanations--i.e., to pursue insights through activities that lead to comprehension.

Several types of cognitive incongruity, or conceptual conflict, are possible. When an individual encounters an event in which a prediction based on prior knowledge is not confirmed, that person is *surprised*, and is motivated to figure out why the prediction failed and to repair his/her knowledge base. On the other hand, if competing ideas appear equally plausible, the person is *perplexed* and seeks out additional information that would assist in selecting one of the alternatives. An awareness that items of knowledge about a topic are not well coordinated or connected brings about a third type of conflict called *discoordination*.

The model predicts that these three types of cognitive incongruity--surprise, perplexity, and discoordination--are not in themselves sufficient to induce comprehension-seeking behavior. Individuals must *recognize* that their comprehension is inadequate and limited. In addition, individuals must believe that they are capable of comprehending the target knowledge in question, and that such comprehension is important and worth the effort and time required. "When subjects experience cognitive incongruity about a target which they value, they are likely to engage in comprehension activity. On the other hand, when they feel cognitive incongruity about a target of little interest or value to them, they will be reluctant to exert the mental effort required for comprehension activity" (Hatano & Inagaki, p. 38).

Finally, according to this model, the expectation of external rewards changes learning goals from comprehension to earning rewards. For this reason, prolonged comprehension activity can be expected only when the individual is not experiencing the need to produce or earn rewards.

Hatano and Inagaki list several instructional strategies for inducing cognitive incongruity. Surprise results when students are asked to make a prediction and are then shown information that contradicts the prediction. Perplexity is often an outcome of class discussions that generate conflicting ideas. Discoordination is experienced when the students' views or explanations are challenged or disputed. All three of these cognitive incongruities are likely to arise during peer interactions, or during teacher-student interactions in which problems are introduced that purposely lead to incongruities.

Hatano promotes dialogue as an effective means of inducing comprehension activity, even when students lack a well-organized knowledge base. "Such interaction (1) tends to produce and amplify surprise, perplexity, and discoordination by helping people monitor their comprehension; and (2) relates the less familiar domain to one's domains of expertise and interest" (Hatano, 1988, p. 61). Social-interactional activities lead participants to seek justifications and explanations more so than when they work alone.

Relevant instructional principles that follow from Hatano and Inagaki's work, together with applications, can be summarized as follows:

1. *Use cognitive incongruities to lead students to examine and extend their understanding of rational numbers. In particular, misconceptions must be acknowledged and addressed.*
Application: Prospective teachers sometimes have very limited knowledge about rational numbers, and their comprehension may be inadequate, even though they are not usually aware of these inadequacies. Students can be made to recognize and deal with inadequacies through problems that produce cognitive incongruities. Setting the stage to induce states of cognitive incongruity should be an important instructional strategy in this type of course. For example, the fact that 1/2 ÷ 1/3 can be modeled with manipulatives, such as pattern blocks, and that the resulting answer can be found without inverting and multiplying often comes as a *surprise* to many students. Situations in which multiplication makes smaller, after having predicted a larger answer, is *perplexing* to many students. *Discoordination* can be the result of the finding that quotients may be thought of as fractions, particularly to students for whom the only known model of fractions is the part-whole area model.

2. *Provide opportunities for dialogue.*
Application: Although class discussions can be classified as dialogue according to Hatano and Inagaki, it frequently happens that some students do not become involved in the discussion, and benefit little from it. Small groups of three or four are more conducive to the type of interactions that can lead to questioning and seeking explanation and justification. Prospective teachers frequently tell us that they have never experienced learning mathematics

in this manner, but rather have had teachers only lecture to them, and then help them individually with assigned homework.

3. *As much as is possible, create a classroom climate free of external rewards.*

Application: A fair grading system is mandatory in this type of course, but different types of evaluation and consequent grading should be attempted when possible. All of the following are consistent with an effort to de-emphasize external rewards: asking the students for self-evaluations, giving written remarks but no numerical or letter grades on homework assignments , group grades for group work, and allowing on take-home quizzes a post-answer discussion within a group or an endorsed check with class notes or the book.

4. *Students need to be convinced that rational number understanding is important and worth the time and effort needed to gain that understanding.*

Application: Knowing that they will soon be teaching these same concepts to children places a value on comprehension that might not otherwise be there. A careful sequencing of problems is needed to convince students who may lack confidence in their ability to understand that they *can* understand. Also, when students in cooperative learning environments begin to take responsibility for one another's learning, they can convince one another that comprehension is possible.

Working toward comprehension may require some special effort. Elliott and Dweck (1988), for example, call attention to two types of learner goals: "(a) performance goals, in which individuals seek to maintain positive judgments of their ability and avoid negative judgments by seeking to prove, validate, or document their ability and not discredit it; and (b) learning goals, in which individuals seek to increase their ability or master new tasks" (p. 5). Naturally one would prefer that prospective teachers in particular would have learning goals. Yet, some data (Dweck, 1987) and much personal experience suggest that many college students are concerned primarily with external performance goals.

Two student-held "implicit theories of intelligence" (cf. Dweck & Leggett, 1988) can also be coupled with these two types of goals. One of these theories holds that intelligence is a fixed entity; the other, that intelligence is changeable. Those who subscribe to the entity theory tend to have performance goals, whereas those who subscribe to the changeable theory tend toward learning goals. Furthermore, entity theorists who perceive their ability as low (a not uncommon view in mathematics among prospective teachers) readily adopt "helpless" response modes. Thus, to establish the desired comprehension set, an effort must be made to combat performance-goal and fixed-intelligence orientations.

Application: Review and discuss with prospective teachers the psychological literature on theories of intelligence and learner goals. Intermittently ask students to reflect on their work with regard to these theories.

Creating An Appropriate Conceptual Environment

In discussing aspects of number sense acquisition, Greeno (1989) used a spatial metaphor to describe a subject-matter domain as a conceptual environment in which individuals learn to live cognitively. Knowing means the ability to find and use, within this environment, the resources needed to understand and reason. When applied to the domain of numbers and quantities, this metaphor "highlights the multilinear and multiconnected nature of knowing in the domain" (p. 46). Individuals with number sense have a sense of quantities and their values that allows effortless movement in the sphere of relations between quantities and operations. They can understand numbers and deal with them in more or less detail, depending upon the context. Just as in a physical environment, there are many things in a conceptual environment that have been previously constructed and placed there, so that learning means developing one's ability to move about and conduct activities, to come to understand the resources, routes, and paths, and occasionally to construct a new path or a new bridge. Thus, each person "constructs a rich set of interconnections among the locations in the space that provides him or her capability of moving around in the environment" (p. 46).

There are two reasons why this metaphor is particularly suited to the planning of rational number instruction for prospective teachers. The first is that the domain in question *is* a domain of quantities and operations, specifically, the field of rational numbers. Prospective teachers traditionally are not familiar with the environment of rational numbers, at least not to the extent that they can move about with ease within that environment. Yet this is our ultimate goal--that prospective teachers truly comprehend the relations between, and operations on, the quantities we call rational numbers, that they can move easily between models and other representations of rational numbers, that they can operate flexibly with rational numbers, that they understand the values associated with rational numbers and can use that understanding to deal with and reason about operations on rational numbers.

The second reason for the efficacy of the metaphor is closely related to the first. Ultimately, we want the *students* of these prospective teachers to have rational number sense also, something that is unlikely to happen if teachers cannot act as guides within this conceptual environment.

> "As learning is analogous to acquiring abilities for finding one's way around in an environment, teaching is analogous to the help that a resident of the environment can give to newcomers. Effectiveness in providing guidance to others is not equivalent to knowing the environment one's self; one can be fully effective in finding and using the resources of an environment, but be of little help to someone else. An effective guide for learners needs to be sensitive to the information they already have, to connect new information to it, to provide tasks and instructions that can be engaged in productively by beginners, to be aware of potential errors that can result from newcomers' partial knowledge, and to help beginners use errors as occasions for learning" (Greeno, 1989, p. 48-49).

Cognitive principles based on this metaphor and applicable to instruction include the following:

1. *Focus on problems, discussion questions, and assignments that lead to increasing number sense, rather than on routine skills and rules for performing these skills.*
Application: Many problems and discussion questions should focus on number magnitude and on estimation tasks, which are believed to increase number sense (Sowder, in press). Students should be continually asked to apply number sense to solving problems. For

example, when give a problem such as 1/2 ÷ 0.5, students should answer without changing the 0.5 to a fraction, and then inverting and multiplying.

2. *Provide opportunities for examination of different problem-solving strategies.* Students should be provided such opportunities in small group work and related assignments where they are asked to contrast their own understandings and learning processes with those of others in their groups. They should then be asked to reflect on the different ways other students approach problems. It must be made clear to students that there are many correct ways of approaching problems, and that there is frequently more that one correct answer.

Current Curricular Recommendations

Not all decisions about a course on rational numbers for prospective teachers can be based solely on research findings. There is also concern that the content of the course be aligned with current recommendations on rational number instruction in schools, since the purpose of the course is to prepare these students to become teachers in tomorrow's schools. Naturally, one would hope that these two sources of information are not in conflict.

In its *Curriculum and Evaluation Standards for School Mathematics* (1989), the National Council of Teachers of Mathematics set forth a vision of mathematical literacy for all students, as well as curricular recommendations for achieving this literacy. The topic of rational numbers is included at several different points in NCTM's *Standards* for grades five through eight. At the middle grades it is recommended that the mathematics curriculum continue to development number and number relationships so that students can "understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and scientific notation) in real-world and mathematical problem situations, develop number sense for whole numbers, fractions, decimals, integers, and rational numbers, ... and investigate relationships among fractions, decimals, and percents" (p. 87). Students should be able to represent rational numbers in a variety of meaningful situations, moving flexibly among concrete, pictorial, and symbolic representations.

The *Standards* further recommend that increased attention be paid to exploring relationships among representations of, and operations on, whole numbers, fractions, decimals, integers, and rational numbers, and to using concrete materials and actively involving students individually and in groups, with decreased attention paid to memorizing rules and algorithms, practicing tedious paper-and-pencil computations, and teaching topics in isolation.

The instructional principles and applications discussed here are consistent with the recommendations put forth in the *Standards*. We believe that a teacher preparation course based on these principles and applications will prepare future teachers to follow the *Standards'* recommendations. We further believe that the instructional principles and applications are general enough to allow alternative interpretations and paths through the content encompassed by rational numbers. While the applications are restricted to rational number learning, the cognitively-based instructional principles are more widely applicable. Within the context of teacher preparation, they can for the most part be applied to any of the content areas studied by prospective teachers, particularly those content areas where some limited learning has already occurred. While we do not argue that these are the only applicable principles that can be gleaned from cognitive research, we believe that they form a good starting point for incorporating the contributions of cognitive science into teacher preparation.

References

Behr, M. J., Wachsmuth, I., & Post, T. R. (1985). Construct a sum: A measure of children's understanding of fraction size. *Journal for Research in Mathematics Education, 16*(2), 120-131.

Behr, M. J., Wachsmuth, I., Post, T. R., & Lesh, R. (1984). Order and equivalence of rational numbers: A clinical teaching experiment. *Journal for Research in Mathematics Education, 15*(5), 323-341.

Berlyne, D. E. (1963). Motivational problems raised by exploratory and epistemic behavior. In S. Koch (Ed.), *Psychology: A study of a science* (Vol. 5, pp. 284-364). New York: McGraw-Hill.

Berlyne, D. E. (1965). *Structure and direction in thinking*. New York: John Wiley.

Carpenter, T. P., Fennema, E., Peterson, P. L., Chiang, C., & Loef, M. (1989). Using knowledge of children's mathematics thinking in classroom teaching: An experimental study. *American Educational Research Journal, 26*(4), 499-531.

Chipman, S. S., Segal, J. W., & Glaser, R. (Eds.). (1985). *Thinking and learning skills: Research and open questions*. Hillsdale, NJ: Erlbaum.

Cobb, P. (1987). Information-processing psychology and mathematics education -- A constructivist perspective. *The Journal of Mathematical Behavior, 6*(1), 3-40.

Confrey, J. (1987). "Misconceptions" across subject matters: Science, mathematics, and programming. In J. D. Novak (Ed.), *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics* (Vol 1, pp. 81-106). Ithaca, NY: Cornell University.

Davis, R. B. (1984). *Learning mathematics: The cognitive science approach to mathematics education*. Norwood, NJ: Ablex.

Dweck, C. S. (1987, April). Children's theories of intelligence: Implications for motivation and learning. Paper presented at the Annual Meeting of the American Educational Research Association, Washington.

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review, 95*(2), 256-273.

Elliot, E. S., & Dweck, C. S. (1988). Goals: an approach to motivation and achievement. *Journal of Personality and Social Psychology, 54*(1), 5-12.

Graeber, A., & Tirosh, D. (1988). Multiplication and division involving decimals: Preservice elementary teachers' performance and beliefs. *The Journal of Mathematical Behavior, 7*(3), 263-280.

Greeno, J. G. (1989). Some conjectures about number sense. In J. T. Sowder & B. P. Schappelle (Eds.), *Establishing foundations for research on number sense and related topics: Report of a conference* (pp. 43-56). San Diego: San Diego State University Center for Research in Mathematics and Science Education.

Greer, B. (1988). Introduction. *Journal of Mathematical Behavior, 7*(3), 193-196.

Hatano, G. (1988). Social and motivational bases for mathematical understanding. In G. B. Saxe & M. Gearhart (Eds.), *Children's mathematics* (pp. 55-70). San Francisco: Jossey-Bass.

Hatano, G., & Inagaki, K. (1987). A theory of motivation for comprehension and its application to mathematics instruction. In T. A. Romberg & D. M. Stewart (Eds.), *The monitoring of school mathematics: Background papers* (Vol. 2, pp. 27-46). Madison, WI: Wisconsin Center for Education Research.

Hiebert, J., & Behr, M. (1988). Introduction: Capturing the major themes. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 1-18). Reston, VA: NCTM, and Hillsdale, NJ: Erlbaum.

Hiebert, J., & Wearne, D. (1985). A model of students' decimal computation procedures. *Cognition and Instruction, 2*(3 & 4), 175-205.

Kerslake, D. (1986). *Fractions: Children's strategies and errors*. Windsor, England: NFER-NELSON.

Kieren, T. E. (1976). On the mathematical, cognitive, and instructional foundations of rational numbers. In R. Lesh & D. Bradbard (Ed.), *Number and measurement: Papers from a research workshop* (pp. 101-144). Columbus, OH: ERIC/SMEAC.

Kieren, T. E. (1988). Personal knowledge of rational numbers: Its intuitive and formal development. In J. Hiebert & M. Behr (Eds.), *Number concepts and operations in the middle grades* (pp. 162-181). Reston, VA: NCTM, and Hillsdale, NJ: Erlbaum.

Lesh, R., Landau, M., & Hamilton, E. (1980). Rational number ideas and the role of representational systems. In R. Karplus (Ed.), *Proceedings of the Fourth International Conference for the Psychology of Mathematics Education* (pp. 50-59). Berkeley, CA: Lawrence Hall of Science.

Linn, M. C. (1986). *Establishing a research base for science education: Challenges, trends, and recommendations* (Report of a National Conference). Berkeley, CA: Lawrence Hall of Science.

National Council of Teachers of Mathematics. (1989a). *Curriculum and Evaluation Standards for School Mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics (1989b). *Professional Standards for Teaching Mathematics, Working Draft*. Reston, VA: Author.

National Research Council. (1989). *Everybody counts: A report to the nation on the future of mathematics education*. Washington, DC: National Academy Press.

Norman, D. A. (1980). Cognitive engineering and education. In D. T. Tuma & F. Reif (Eds.), *Problem solving and education: Issues in teaching and research* (pp. 97-107). Hillsdale, NJ: Erlbaum.

Novak, J. D. (Ed.). (1987). *Proceedings of the Second International Seminar on Misconceptions and Educational Strategies in Science and Mathematics*. Ithaca, NY: Cornell University.

Novillis, C. F. (1976). An analysis of the fraction concept into a hierarchy of selected subconcepts and the testing of the hierarchical dependencies. *Journal for Research in Mathematics Education, 7*(3), 131-144.

Novillis-Larson, C. (1980). Locating proper fractions on number lines: Effect of length and equivalence. *School Science and Mathematics, 53*(5), 423-428.

Peck, D. M., & Jencks, S. M. (1981). Conceptual issues in the teaching and learning of fractions. *Journal for Research in Mathematics Education, 12*(5), 339-348.

Peterson, P. (1988). Teachers' and students' cognitional knowledge for classroom teaching and learning. *Educational Researcher, 17*(5), 5-14.

Post, T. R., Harel, G., Behr, M. J., & Lesh, R. (1988). Intermediate teachers' knowledge of rational numbers concepts. In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating research on teaching and learning mathematics* (pp. 194-217). Madison, WI: Wisconsin Center for Education Research.

Radatz, H. (1980). Students' errors in the mathematical learning process: A survey. *For The Learning of Mathematics, 1*(1), 16-20.

Resnick, L. B. (1986). The development of mathematical intuition. In M. Perlmutter (Ed.), *Perspectives on intellectual development: The Minnesota Symposia on Child Psychology* (Vol 19, pp. 159-194). Hillsdale, NJ: Erlbaum.

Resnick, L. B., & Omanson, S. F. (1987). Learning to understand arithmetic. In R. Glaser (Ed.), *Advances in instructional psychology* (Vol. 3, pp. 41-95). Hillsdale, NJ: Erlbaum.

Sackur-Grisvard, C., & Leonard, F. (1985). Intermediate cognitive organizations in the process of learning a mathematical concept: The order of positive decimal numbers. *Cognition and Instruction, 2*(2), 157-174.

Scardamalia, M., Bereiter, C., McLean, R. S., Swallow, J., & Woodruff, E. (1989). Computer-supported intentional learning environments. *Journal of Educational Computing Research, 5*(1), 51-68.

Shaughnessy, J. M. (1985). Problem solving derailers: The influence of misconceptions on problem-solving performance. In E. A. Silver (Ed.), *Teaching and learning mathematical problem solving: Multiple research perspectives* (p. 399-415). Hillsdale, NJ: Erlbaum.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*(1), 1-22.

Silver, E. A. (1981). Young adults' thinking about rational numbers. In T. R. Post & M. P. Roberts (Eds.), *Proceedings of the Third Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 149-159). Minneapolis: University of Minnesota.

Sowder, J. T. (in press). Making sense of numbers in school mathematics. In G. Leinhardt, R. Putnam, and R. Hattrup, (Eds.), *Analysis of Arithmetic for Mathematics*. Hillsdale, NJ: Erlbaum.

Sowder, J. T., Crosswhite, F. J., Greeno, J. G., Kilpatrick, J., McLeod, D. B., Romberg, T. A., Springer, G., Stigler, J. W., & Swafford, J. O. (1989). Setting a research agenda. Reston, VA: NCTM and Hillsdale, NJ: Erlbaum.

Sowder, J. T., & Markovits, Z. (1989). Effects of instruction on number magnitude. In C. A. Maher, G. A. Goldin, & R. B. Davis (Eds.), *Proceedings of the Eleventh Annual Meeting: North American Chapter of the International Group for the Psychology of Mathematics Education* (pp. 105-110). New Brunswick, NJ: Center for Mathematics, Science, and Computer Education, Rutgers.

Sowder, L. (1988). Children's solutions of story problems. *Journal of Mathematical Behavior,* 7(3), 227-238.

APPENDIX B.2

POSTER SESSION FOR PME

A COGNITIVE APPROACH TO INSTRUCTION FOR PROSPECTIVE TEACHERS

Nadine Bezuk, Judith Sowder, and Larry Sowder
San Diego State University

Cognitive science research on intentional learning and motivated comprehension was combined with cognitively based research on rational numbers and probability and statistics in designing a mathematics course for prospective elementary teachers. Plans for instruction and student evaluation based on these research findings are discussed.

This paper presents an account of the planning of a mathematics course for preservice elementary teachers, in which principles derived from cognitive science research provided the foundation for curricular and instructional planning decisions. The course is offered in a mathematics department, and until this revision, has been taught in a quite traditional fashion, using a standard text on mathematics for elementary teachers. As the second in a sequence of three required courses, this course focuses on rational numbers, probability, and statistics.

Background

Much of the cognitive psychological research has been done in the areas of mathematics and science learning, and has contributed to our understanding of learning in these subject areas. Linn (1986) has pointed out that science (including mathematics) education would be strengthened by building upon what we already know about the cognitive structure of the subject matter. The advances made in cognitive research over the past decade in formulating and testing models of the kinds of information structures students acquire when learning, particularly when these models are applied to students' performance on mathematical tasks, have made it possible for us to gain insights into the development of mathematical ideas (Sowder, Crosswhite, Greeno, Kilpatrick, McLeod, Romberg, Springer, Stigler, & Swafford, 1989). It is time to incorporate relevant research findings from cognitive psychology into teacher preparation courses.

Shulman (1987) has pointed out that such research-based knowledge is at the very heart of his definition of needed pedagogical content knowledge. He suggests that prospective teachers need a solid grounding in the content areas they will teach and also must examine that content in terms of its structure, organization, important ideas, and skills. Pedagogical content knowledge "includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning" (Shulman, 1986, p. 9).

Many cognitive researchers view learners as active constructors of their own knowledge. One aspect of constructivist research is the study of student preconceptions and misconceptions in mathematics. Not until teachers are aware of the student conceptions that interfere with a full understanding of fundamental ideas in mathematics and science, can they create instructional conditions that allow for change and transformation of those conceptions. Yet those of us who teach teachers know that elementary teachers themselves frequently harbor the same misconceptions and misunderstandings as their students, particularly when problems are not straightforward and algorithmic in nature. The misconceptions formulated in their own childhoods are frequently carried with teachers into the classrooms in which they teach. Psychological research on memory organization and retrieval does not offer much hope for the "unlearning" of misconceptions. Rather, students must form new concepts, easily accessible through a richly connected network. Although the old conceptions are not replaced, they become more difficult to access than the new ones, and are therefore less likely to be the ones called into short-term memory.

Entering prospective teachers have acquired a vast body of procedural knowledge, particularly about rational numbers. The most adept have what Hatano (1988) refers to as routine expertise, that is, they are able to carry out procedures quickly and accurately. While routine expertise might be sufficient for most day-to-day work, it is not sufficient for prospective teachers. To be effective, a teacher must understand how and why a given procedure works, how it can be adapted to new situations, and when it can be abandoned in favor of another procedure, that is, a teacher must have adaptive expertise (Hatano, 1988).

A Cognitive Approach to Instructional Planning

Two sources in particular offered us assistance in formulating instructional guidelines that fit within a cognitive science framework and that were based on cognitive research. These two sources are discussed here.

Intentional Learning

Scardamalia, Bereiter, McLean, Swallow, & Woodruff (1989) have developed a set of learning principles based on cognitive research. They show that successful learners "use a variety of cognitive strategies and self-management procedures to pursue knowledge-related goals, to relate information to old, to monitor their understanding, to infer unstated information, and to review, reorganize, and reconsider their knowledge" (p. 53). In contrast, the learning of those who are less successful is characterized as additive rather than transformational, as focusing on surface features, and as organized around topics rather than around goals. The authors note the persistence of immature strategies which have led students to partial success, even though they are inadequate for all situations.

Based on experiments that led to active teaching strategies, Scardamalia et al. advocate a teaching approach based on modeling, explanation of strategies, and encouragement of students to take greater responsibility for their own learning. In particular, they advocate a theory-based instructional approach they call procedural facilitation which provides support while learners try to adopt more complex strategies. "These supports include turning normally covert processes into overt processes; reducing potentially infinite sets of choices to limited, developmentally appropriate sets; providing aids to memory; and structuring procedures so as to make it easier to escape from habitual patterns" (p. 54). The supports are designed to lead eventually to independent processing by learners.

Enhancing Motivation for Comprehension

Working within the framework of cognitive instructional psychology, Hatano and Inagaki (1987) have constructed a model for motivation for comprehension. In this model, the individual in a state of cognitive incongruity is aware that comprehension is inadequate but is within reach, and becomes motivated to seek satisfactory explanations, that is, to pursue insights through activities that lead to comprehension.

The model predicts that cognitive incongruity is not in itself sufficient to induce comprehension-seeking behavior. An individual must realize that their comprehension of a topic is inadequate and limited. In addition, the individuals must believe that they are capable of comprehending the target knowledge in question, and that such comprehension is important and worth the effort and time required. "When subjects experience cognitive incongruity about a target which they value, they are likely to engage in comprehension activity. On the other hand, when they feel cognitive incongruity about a target of little interest or value to them, they will be reluctant to exert the mental effort required for comprehension activity" (p. 38).

Finally, according to this model, the expectation of external rewards changes learning goals from comprehension to earning rewards. For this reason, prolonged comprehension activity can be expected only when the individual is not experiencing the need to produce or earn rewards.

Hatano and Inagaki list several instructional strategies for inducing cognitive incongruity. For example, students can be asked to make a prediction and then are shown information that contradicts the prediction. Class discussions can generate conflicting ideas that can be discussed. Students' views or explanations may be challenged or disputed. All of these instances are likely to arise during peer interactions, or during teacher-student interactions in which problems are introduced that purposely lead to incongruities.

Cognitively Based Research on Rational Number Learning

Much of the research on rational number learning has examined students' existing understandings of rational number concepts and operations. The Rational Number Project (Behr, Lesh, Post, & Silver, 1983) has investigated these understandings in great detail. They report that students often are distracted by irrelevant features of rational number tasks, particularly when students are in the process of refining their understandings of a concept, and that instruction often promotes rote and procedural rather than rich, conceptual knowledge of rational numbers.

Sackur-Grisvard and Leonard (1985) note the existence of several stages in students' learning of the concept of ordering decimal numbers, which they believe are stable intermediate organizations developed as children are in the process of learning the concept of ordering decimals. Hiebert and Wearne (1985) report that students' decimal computation procedures are often syntactically, rather than semantically, based.

Although most rational number research to date has centered on school-aged children, an emerging body of research is finding similar weaknesses in elementary-school teachers' understandings. Post, Harel, Behr, and Lesh (1988) found compound difficulties regarding classroom teachers' understandings of rational numbers. Not only did many teachers have difficulty in correctly answering conceptual and computational rational number questions, many also were unable to explain their solution processes in more than a procedural manner, even though they were asked to explain their solution to problems as if they were explaining them to a child.

Virtually all research on rational number concepts have made similar conclusions: most students (a) are developing only rote, procedural knowledge, rather than rich understandings, (b) focus on syntactic rather than semantic rules, (c) prefer only one interpretation of rational numbers, the part-whole interpretation, often using a circular model, and (d) often have difficulty using models to illustrate operations or to connect operations on objects with symbolic algorithms.

Cognitively Based Research on Probability and Statistics

There is a rich body of research literature on the judgmental heuristics and biases used by people when answering probability questions (see Shaughnessy, in press, for a review). Most often the use of these heuristics, such as judging estimates of likelihood on the basis of representativeness or availability, or treating weighted means as simple means, reveal deep-rooted and serious misconceptions. Unless the more common misconceptions are personally examined and overcome, prospective teachers will be unable to teach probability and statistics effectively .

There are two other serious instructional problems. According to Konold (1986), "Basing a definition of probability on equally likely events is both circular and limits the extension of probability to an unimaginative range of phenomena" (p. 345). The pedagogical question of whether to begin with a relative frequency approach or a relative proportion model is a difficult one. It is doubtful that there *is* a right way, and the relationships between the two methods need to be explored and discussed. The second serious problem deals with the notion of randomness. Can students understand probability unless they first understand randomness? But can they understand randomness outside the context of probability? Again, there is no clear answer, but certainly the exploration of the notion of randomness deserves more time than it now receives in courses for prospective teachers.

Plans for Instruction

The major goal of this course is to lead students not only to a better understanding of rational numbers and stochastics, but to be able to examine the limitations and errors of their prior understandings, and make the necessary changes. The work by Scardamalia et al. includes several principles that we used to guide our instructional planning. By making knowledge-construction activities overt, students can be made aware of when they are connecting old and new knowledge, and when they are solving problems based on understanding rather than on procedural knowledge. By maintaining attention to cognitive goals, students will be given opportunities to sort through what they did and did not understand. Cooperative learning in groups will allow students to provide each other with process-relevant feedback, and will encourage maximum use and examination of existing knowledge. When students realize that their comprehension is incomplete, they will be able to benefit from lessons organized around problems selected to induce cognitive incongruity.

Plans for Student Evaluation

Coping with students' desire for external awards is one of the most difficult tasks within the area of assessment. But Hatano and Inagaki (1987) suggest that such external rewards will interfere with students' drive for comprehension. Asking students for reflective self-evaluations, making them responsible for the learning of others within their groups, and examining errors in an open and supportive fashion might make "grades" somewhat less important.

Adaptive expertise must also be evaluated. Nonroutine tasks will be given in homework, in group work, and in tests. Examining students' written reflections of their learning will also give some indication of adaptive expertise. Interviews will be used to evaluate this course and the development of students' understanding. A major limitation of individual interviews, however, is that they are difficult to use in a non-research setting because of time constraints and scheduling difficulties.

Final Comment

The planning for this course will continue throughout the instructional period. One of the major questions to be examined will be whether such a course can be easily taught by someone unfamiliar with the research literature and rationale for instructional sequences. This is an important question, because many instructors of this course both here and elsewhere are trained only in mathematics, and not in pedagogy or research.

References

Behr, M. J., Lesh, R., Post, T. R., & Silver, E. A. (1983). Rational number concepts. In R. Lesh & M. Landau (Eds.), *Acquisition of Mathematics Concepts and Processes*, (pp. 91-126). NY: Academic Press.

Hatano, G. (1988). Social and motivational bases for mathematical understanding. In G. B. Saxe and M. Gearhart (Eds.), *Children's Mathematics*. New Directions in Child Development, no. 41. San Francisco: Jossey-Bass.

Hatano, G., & Inagaki, K. (1987). A theory of motivation for comprehension and its application to mathematics instruction. In T. A. Romberg & D. M. Stewart (Eds.), *The monitoring of school mathematics: Background Papers*. Vol. 2: Implications from Psychology; Outcomes of Instruction. Program report 87-2. Madison: Wisconsin Center for Educational Research.

Hiebert, J. & Wearne, D. (1985). A model of students' decimal computation procedures. *Cognition and Instruction, 2* (3 & 4), 175 - 205.

Konold, C. (1986). Probability is conceptually different. In G. Lappan and R. Even (Eds.), *Proceedings of the Eighth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education (PME-NA)*. East Lansing, Michigan: PME-NA, 342 - 346.

Linn, M. C. (1986). *Establishing a Research Base for Science Education: Challenges, Trends, and Recommendations* (Report of National Conference). Berkeley, CA: National Science Foundation.

Post, T. R., Harel, G., Behr, M. J., & Lesh, R. (1988). "Intermediate Teachers' Knowledge of Rational Number Concepts". In E. Fennema, T. P. Carpenter, & S. J. Lamon (Eds.), *Integrating Research on Teaching and Learning Mathematics*. Madison, WI: National Center for Research in Mathematical Sciences Education, 194 - 217.

Sackur-Grisvard, C., & Leonard, F. (1985). Intermediate cognitive organizations in the process of learning a mathematical concept: The order of positive decimal numbers. *Cognition and Instruction, 2* (2), 157 - 174.

Scardamalia, M., Bereiter, C., McLean, R. S., Swallow, J., & Woodruff, E. (1989). Computer-supported intentional learning environments. *Journal of Educational Computing Research, 5*(1), 51 - 68.

Shaughnessy, J. M. (in press). Research on probability and statistics: Reflections and directions. In D. Grouws (Ed.), *Handbook for Research in Mathematics Education*. New York: Macmillan.

Shulman, L. S. (1987). Knowledge and teaching: Foundations of the new reform. *Harvard Educational Review, 57*, 1 - 22.

Shulman, L. S. (1986). Those who understand: Knowledge growth in teaching. *Educational Researcher, 15*, 4 - 14.

Sowder, J. T., Crosswhite, F. J., Greeno, J. G., Kilpatrick, J., McLeod, D. B., Romberg, T. A., Springer, G., Stigler, J. W., & Swafford, J. O. (1989). *Setting a Research Agenda*, volume 5 in the *Research Agenda for Mathematics Education*. Reston, VA: Lawrence Erlbaum Associates and National Council of Teachers of Mathematics.

The preparation of this manuscript was supported by the National Science Foundation Grant No. TPE 50315. The opinions expressed here do not necessarily reflect the position, policy, or endorsement of the National Science Foundation.

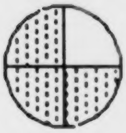
APPENDIX B.3

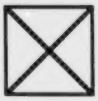
MATH 211

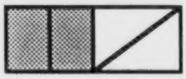
SELECTED GROUP ACTIVITIES, ASSIGNMENTS, QUIZZES

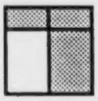
Assignment for Lesson I

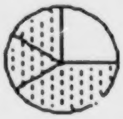
1. Which of the following represent the fraction $\frac{3}{4}$? For those which do not, tell why not.

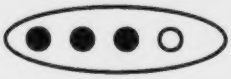

a.

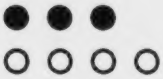

b.

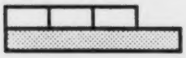

c.

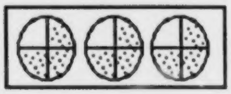

d.

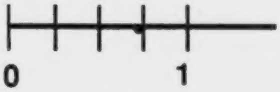

e.


f.

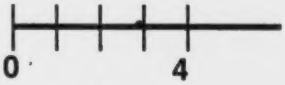

g.


h.

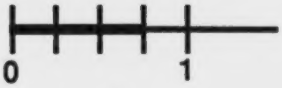

i.



j.



k.



l.

75%	$3 \div 4$	.75	.74	$\frac{1\frac{1}{2}}{2}$
m.	n.	o.	p.	q.

2. a) Mentally take a 3 decimeter strip of paper, fold it end to end, fold it again, so that you have 4 equal parts. Is each part more than or less than one decimeter?_____

If the length of each part is x, write an equation using x, 3, and 4. ____________

How long is each part?______________

b) Repeat this procedures using a 6 decimeter strip, folding it three times end-to-end instead of two times:

What is the equation this time?____________________

Tell about the folded parts of the two strips.________________

If you have any difficulty with this problem, use strips and do the folding. Can any two strips, where one is twice the length of the other, be used?

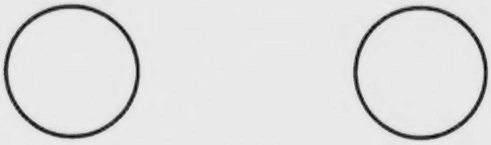
3. Can you find a fraction between each of these pairs? If so, give at least one.

a) 0 and $\frac{1}{4}$? ____________ b) $\frac{4}{8}$ and $\frac{7}{8}$? ____________

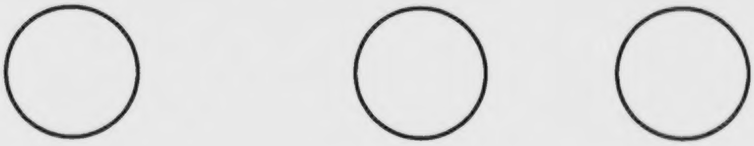
c) $\frac{4}{8}$ and $\frac{5}{8}$? ____________ d) $\frac{2}{5}$ and $\frac{3}{5}$? ____________

e) $\frac{3}{4}$ and $\frac{3}{5}$? ____________

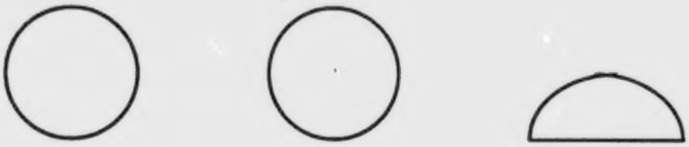
4. a) Demonstrate with a drawing how 2 large pizzas can be divided fairly among 5 people. How much does each person get?



b) Demonstrate with a drawing how 3 large pizzas can be divided fairly among 5 people. How much does each person get?



c) Demonstrate with a drawing how $2\frac{1}{2}$ large pizzas can be divided fairly among 5 people. How much does each person get?



d) Discuss how this relates to problem 3d.

Name________________________

Assignment page to be turned in at the beginning of the next class.

1. Were there any problems/answers in the group exercises where there was disagreement? If so, which ones, and what was the basis for the disagreement?

2. Reflect back on the class activities, discussions, and this assignment.

   a) Is this what you would expect from a first lesson on fractions? Why or why not?

   b) Did your thinking about the group problems differ from others' in the group? If so, in what ways?

   c) Are there any problems on this assignment that caused you confusion? Which, and why?

Group Activity #2 on Fractions

Directions: Work through these problems as indicated. Put your answers on these pages. Then do the group report. Make sure you agree on the rules that are put on your group report.

1. Using the intersecting circles, find some numbers

   close to 0: ______________________

   close to 1: ______________________

   close to $\frac{1}{2}$: ______________________

2. Together, formulate some general rules:

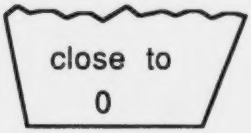
   A fraction is close to 0 when:

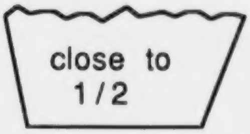
   A fraction is close to $\frac{1}{2}$ when:

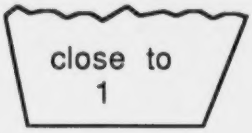
   A fraction is close to 1 when:

3. Alone, draw lines from the fractions to the baskets, then compare with your group. Discuss any differences.

   $\frac{4}{7}$, $\frac{2}{9}$, $\frac{11}{12}$, $\frac{1}{3}$, $\frac{17}{35}$, $\frac{15}{34}$, $\frac{11}{108}$, $\frac{3}{12}$, $\frac{9}{8}$







4. Now, formulate more specific rules:

A fraction is less than $\frac{1}{2}$ when:

A fraction is more than $\frac{1}{2}$ when:

A fraction is less than 1 when:

A fraction is more than 1 when:

A fraction is more than 0 when:

5. Can a fraction be less than 0? If not, why not. If so, when?

6. Using the circles again, find some fractions close to $\frac{1}{3}$: ________________

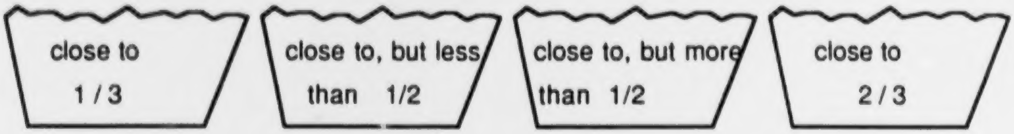
close to $\frac{2}{3}$: ________________

When is a fraction close to $\frac{1}{3}$?

When is a fraction close to $\frac{2}{3}$?

7. Alone, draw lines from fraction to baskets, then compare your diagrams. Discuss and resolve any differences.

$\frac{4}{9}$, $\frac{21}{29}$, $\frac{49}{99}$, $\frac{15}{35}$, $\frac{27}{54}$, $\frac{42}{63}$, $\frac{31}{45}$, $\frac{22}{47}$, $\frac{99}{102}$, $\frac{2}{9}$


close to 1 / 3
close to, but less than 1/2
close to, but more than 1/2
close to 2 / 3

8. Using your new knowledge of "close to" for fractions, see if you can do these problems without using your old rules for comparing and operating on fractions.

Circle the larger of:

(a) $\frac{3}{4}$ and $\frac{4}{9}$ (Hint: Compare both to $\frac{1}{2}$)

(b) $\frac{1}{3}$ and $\frac{1}{4}$ (Hint: Which is closer to 0?)

(c) $\frac{3}{4}$ and $\frac{9}{10}$

(d) $\frac{2}{5}$ and $\frac{3}{6}$

(e) $\frac{7}{12}$ and $\frac{5}{8}$

Find a fraction between:

(f) $\frac{2}{5}$ and $\frac{3}{5}$

(g) $\frac{10}{33}$ and $\frac{5}{12}$

Estimate and discuss your estimates

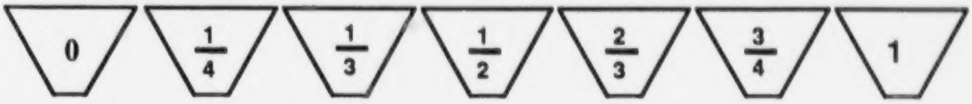
(h) $\frac{7}{8} + \frac{9}{10}$ (k) $\frac{7}{12} + \frac{5}{8}$

(i) $\frac{15}{16} - \frac{1}{7}$ (l) $1\frac{3}{4} - \frac{9}{10}$

(j) $\frac{20}{31} - \frac{1}{3}$

9. Draw lines to show what these decimal numbers and percents are close to: (Don't use a calculator unless you have to.

.29 .52 94% 32% .78 .49 24% 5% 70%



10. Use your "close to" knowledge to estimate the following"
The first one is done for you.

(a) .49 x 102 is about $\frac{1}{2}$ of 100, or 50.

(b) 32% of 12

(c) 94% of 500

(d) 0.52 x 789

(e) 23% of 81

(f) 35% of 22

(g) 76% of $210

Name a recorder and complete the group activity report to be turned in.

Report on Group Activity #2: To be turned in at the end of class.

Group number:

1. How do you know when a fraction is more than $\frac{1}{2}$?

2. How do you know when a fraction is close to $\frac{1}{3}$?

3. How do you:

   (a) Estimate $\frac{7}{12} + \frac{5}{8}$?

   (b) Estimate 0.52 x 789?

4. Were there any problems in the group activity where you disagreed? Is so, which ones?

Signed by group members: ____________________

____

____

____

____

Math 211 **Name**________________________

Take-home Quiz, February 16

1. Illustrate three different ways of finding fractions between $\frac{5}{11}$ and $\frac{6}{11}$.

Find a fraction between 0.078 and 0.079

2. Are the whole numbers dense? Why or why not?

Are the fraction numbers dense? Why or why not?

Are the decimal numbers dense? Why or why not?

3. When asked: "What is a fraction" two students gave these answers: "It is part of a whole" and the other said "No, it is a way of representing division." How would you respond?

4. Explain when a fraction is close to $\frac{1}{3}$.

5. In each case, explain how to chose the larger, without a calculator.

(a) $\frac{3}{8}$ and $\frac{7}{12}$

(b) $\frac{1}{8}$ and $\frac{1}{6}$

(c) $\frac{13}{14}$ and $\frac{14}{13}$

(d) $\frac{7}{8}$ and $\frac{8}{9}$

6. Place the following numbers on the number line, by using the letter that goes with the number:

$A = \frac{1}{2}$, $B = \frac{2}{3}$ $C. = \frac{11}{15}$ $D = 0.68$ $E = 0.6$ $F = \frac{7}{13}$ $G = \frac{11}{14}$

7. Estimate the following, and in each case <u>explain how</u> you got your estimate.

(a) .23 x 798

(b) $\frac{1}{9}$ x 34

(c) $3\frac{1}{4} \div \frac{7}{8}$

(d) $2\frac{1}{7} + 13\frac{2}{15}$

8. This array represents $\frac{4}{9}$ of a whole.
Show the whole.

```
# #     #
          ##  ##   #  #
                 #   ##
```

9. If a friend said : "I think 4.28 is larger than 4.3 because 28 is larger than 3" how would you respond?

Group Activity #3 on Fraction and Decimal Numbers

NOTE: You will need to use fraction tiles on Part II of this assignment. There are only two boxes of tiles. Groups 4 and 5 should begin Part II immediately, and when they finish, other groups may use the tiles.

Part 1: The Number Line Model for fraction and decimal numbers.

1. Individually, put these numbers where they belong on the number line. Then compare answers and clear up any discrepancies.

$$\frac{1}{10},\ \frac{4}{3},\ \frac{8}{9},\ \frac{5}{8},\ \frac{11}{16},\ \frac{3}{4},\ \frac{7}{8},\ \frac{9}{5},\ \frac{1}{16},\ 1\frac{1}{2},\ \frac{1}{16},\ \frac{3}{11},\ \frac{1}{2}$$

2. Draw number lines for each of these, and show the number. Compare.

$\frac{1}{4}$ of 1, $\frac{1}{4}$ of 3, $\frac{1}{4}$ of 2, $\frac{1}{3}$ of 2, $\frac{2}{3}$ of 4

3. Place these numbers in order from smallest to largest. Make sure you agree.

0.781, $\frac{3}{4}$, 0.718, 0.7, $\frac{11}{16}$, $\frac{16}{11}$, 0.7

Part II: Part-whole model, using area (NEED: Fraction tiles.)

1. If the clear square tile is one whole, what do the other tiles represent? Are there different ways to figure this out?

2. Suppose, instead, the gold tile is one whole. Now what do the other tiles represent?

3. Use the tiles to show whether <, >, or = should be placed between:

$\frac{11}{16}$ and $\frac{5}{8}$ $\frac{11}{16}$ and $\frac{2}{3}$ $\frac{2}{3}$ and $\frac{5}{8}$ $\frac{2}{3}$ and $\frac{4}{6}$

4. A fourth-grader told me: "2/3 is larger than 5,8 because 5/8 is 1/8 away from 1/2 and 2/3 is 4/6, so it is 1/6 away from 1/2, and 1/8 is smaller than 1/6." Analyze and discuss this child's reasoning. Check it out with tiles.

Part III: Part-whole model, with discrete objects (Note: Counters needed)

1. Using red and blue counters, represent 2/3. Can this be done in more than one way?

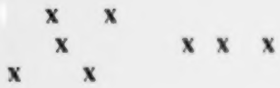
2. Using red and blue counters, represent 8/12. Compare with the above answers.

3. Find 1/2 of 12 counters, _______ ,3/7 of 14 counters _______, 0.5 of 8 counters _______, 0.2 of 5 counters _______, 1/3 of 8 counters ________. Some students say that it is not possible to find 0.2 of 5 counters. Are they right? If so, why? If not, why do you suppose they say that?
Answer the same questions for 1/3 of 8 counters.

4. Put out 4 yellow counters and 8 green counters. Write out all the sentences you think of. For example: There are half as many yellows and greens.

Part IV Some problems for you.....

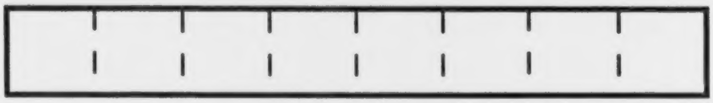
1. If this much represents 3/7, what is the unit (or the whole)?



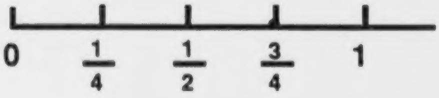
2. What happens to 9/8 if the numerator is tripled and the denominator is divided by 4? Does it get larger? Smaller? Stay the same?

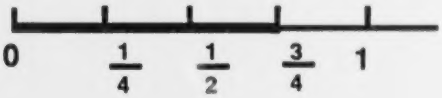
3. What is the rule for finding equivalent fractions? Why does it work?

4. This stick of margarine was marked into 8 equal parts by the manufacturer. This is enough for $1\frac{1}{2}$ recipes of cookies. Show the part needed for one recipe. How much of a stick did you show?



5. When asked to represent $\frac{3}{4}$ on the number line, two students gave these answers. Are they both correct? Do they both use the number line model? Discuss!





Make up some problems like these and solve them.

MATH 211: Assignment for February 19

Part I: Review

1. Review meanings for addition, subtraction (take-away, missing addend, comparison), multiplication (repeated addition, rectangular array, and Cartesian product), and division (sharing, and repeated subtraction), given on pages 62, 68, 69 (top and 2/3 down the page), 75, 76, 81, 82, and 83. (The section on division needs to be read entirely to find the two meanings.) For each of these situations, tell what operation(s) and meanings apply. The first two are done for you.

(a) Janice is going on a 270 mile trip. Her car gets 30 miles to the gallon of gasoline. How much gasoline will she need?

Solution: How many 30s are in 270? That is, how many times can 30 be subtracted from 270. This uses the repeated subtraction meaning for division.

(b) Bill has $38. He needs $55 to buy a new tire for his car. How much more does he need?

Solution: What number needs to be added to 38 to get 55? To find out, I need to subtract. This is an example of the missing addend meaning for subtraction.

(c) In January, Gail was 121.5 centimetres tall. In October, she was 130 centimetres tall. How much had she grown? (There are two defensible answers.

(d) One kind of germ is 0.08 centimetres long. How many of the germs would it take to made 4 centimetres?

(e) One job gives an automatic cost-of-living raise. On a $7-an-hour job, what would be the new pay after a 4% cost-of-living raise?

(f) Seven of the 35 children in the class had chicken pox. What percent of the children is that?

(g) One class had 32 children. Three-fourths of the children got to go on an overnight trip. How many children went on the trip?

(h) As it happens, the 560 children in one school are in 16 classes, all of the same size? How many children are in each class?

(i) A scooter got 144 miles per gallon of gasoline. How far could the scooter go on 0.9 gallons?

(j) You want to give a total of $78 to 2 charities. If you give each of them the same amount, how much would each get?

(k) You have a total of $78. How many $2 could you get for it?

(l) By the time you have jogged 2.3 miles, your friend has jogged 3.5 miles. How far behind are you?

Part II: Exploration

1. It is sometimes easier to mentally compute with fractions than with decimals, and it is therefore handy to have some fraction benchmarks for decimals. For example: To estimate 0.47 x 395, I can think of 0.47 as almost 1/2, and 395 as almost 400, so a good estimate would be 1/2 of 400, or 200.For each of the following, find fraction benchmarks to complete the problem.

(a) Estimate: 0.74 of 789

(b) Estimate: 0.70 of 303

(c) Estimate: 0.31 of 28

(d) Estimate: 4.26 x 19

(e) Estimate: 3.77 - 2.28

2. In lesson 1, you learned to think of 2/3 as 2÷3. You drew pictures of 2 pizzas divided up among 3 people. Using this same notion, what is the meaning of 0/3? of 3/0?

3. Order these sets of numbers from smallest to largest, without a calculator.

(a) 0.045, 0.03, 0.002, 0.111, 0.009, 0.28

(b) 0.36, $\frac{5}{8}$, $\frac{6}{9}$, 0.666, 0. 4, 0.2, $\frac{1}{3}$, 0.3

Math 211
Group Activity #5

1. For each problem, first choose an interpretation (combining for addition; take-away, missing-addend, or comparison for subtraction), second, act out the problem with pattern block and record the solution, and third, write a story problem for your interpretation. (Try using all three subtraction interpretations in c through f.)

(a) $3\frac{1}{2} + 2\frac{1}{3} = n$

(b) $\frac{2}{3} + \frac{5}{6} = n$

(c) $4 - 2\frac{1}{3} = n$

(d) $1\frac{1}{2} - \frac{5}{6} = n$

(e) $\frac{1}{2} - \frac{1}{3} = n$

(f) $3\frac{1}{2} - \frac{3}{4} = n$

2. Two students were asked to solve $4\frac{1}{4} - 2\frac{1}{3}$. Here are their solutions:

Student 1: $4\frac{1}{4} - 2\frac{1}{3} = \frac{17}{4} - \frac{7}{3} = \frac{51}{12} - \frac{28}{12} = \frac{23}{12} = 1\frac{11}{12}$

Student 2: $4\frac{1}{4} - 2\frac{1}{3} = 4\frac{3}{12} - 2\frac{4}{12} = 3\frac{15}{12} - 2\frac{4}{12} = 1\frac{11}{12}$

(a) Discuss these solutions.

(b) Work the problem in #1 algorithmically, just for practice. If time permits try them in the calculator.

3. Try these problems mentally. Discuss the strategies you need.

(a) $\frac{3}{4} + \frac{5}{6} + \frac{1}{4}$

(b) $\frac{3}{8} + \frac{1}{2}$

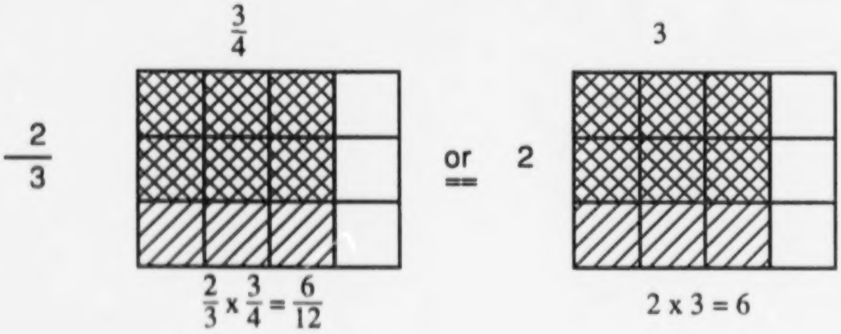
(c) $6\frac{3}{4} - 3\frac{1}{2} - 1\frac{1}{4}$

(d) $2\frac{2}{3} + 5\frac{1}{6} + 3\frac{1}{6} - 4$

(e) $7 - 2\frac{3}{16}$ (Think: What do I need to add to $2\frac{3}{16}$ to get 7.)

(f) $\frac{4}{9} \times \frac{3}{7} \times \frac{9}{4}$

4.. The area model is almost always used to illustrate multiplication of fractions. Problem #6 in Group Activity #4 can also be thought of as:



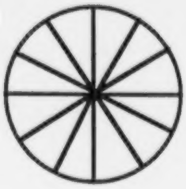
(a) What is the unit (whole) in each case?

(b) What model(s) are used in #1 on Group Activity #4?

(c) Is the Cartesian Product ever appropriate for fraction multiplication?

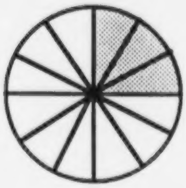
Math 211
Assignment - following Group Activity #5 **Name:**__________________

1. (a)



Shade in $\frac{1}{4}$, then shade in $\frac{1}{3}$. What fraction of the whole is shaded in?

(b)



$\frac{1}{4}$ is already shaded. Cross-hatch $\frac{1}{3}$ of what is shaded in. What fraction of the whole is shaded?

(c) Why are the answers different?

2. Given 6394 ÷ 12, invent stories that will yield each of the following solutions:

(a) 532

(b) 533

(c) 532 remainder 10

(d) $532\frac{5}{6}$

(e) 532.84 remainder 4

(f) 532.8333....

(g) About 530

3. Try these problems alone mentally. Later in your group discuss the strategies you used.

(a) $\frac{3}{4} + \frac{5}{6} + \frac{1}{4}$

(b) $\frac{3}{8} + \frac{1}{2}$

(c) $6\frac{3}{4} - 3\frac{1}{2} - 1\frac{1}{4}$

(d) $2\frac{2}{3} + 5\frac{1}{6} + 3\frac{1}{6} - 4$

(e) $7 - 2\frac{3}{16}$ (Think: What do I need to add to $2\frac{3}{16}$ to get 7.)

(f) $\frac{4}{9} \times \frac{3}{7} \times \frac{9}{4}$

Also: Musser & Burger text, pages 194-199: Part A: 1, 2, 3, 25, 27, 31, 32, 35
Part B: #2 a, c, 5, 6, 8, 31, 34

Turn in at the beginning of the next class, on a clean page:

(1) What new things have you learned about fractions in these last two group activities?

(2) How confident are you about your understanding of fractions?
How has this changes since the beginning of the course?

(3) Can you describe one or two things you learned in groups that you might not have learned had you worked alone?

Math 211
Group Activity # 6: Using the Explorer

TASK: Maximize the result of the following problems:

1. Using the digits 3, 4, 5, 6, 7, & 8:

A. $\square + \frac{\square}{\square} =$

B. $\square - \frac{\square}{\square} =$

C. $\square \times \frac{\square}{\square} =$

D. $\square \div \frac{\square}{\square} =$

E. Is it possible to make a generalization about the size of the result obtained when dividing a whole number by a value larger than 1 (e.g., is the result always smaller or larger than the original number)? Explain.

F. Is it possible to make a generalization about the size of the result obtained when dividing a whole number by a value smaller than 1 (e.g., is the result always smaller or larger than the original number)? Explain.

2. Using the digits 3, 4, 5, 6, 7, & 8:

A. $\square + \frac{\square}{\square} =$

B. $\square - \frac{\square}{\square} =$

C. $\square \times \frac{\square}{\square} =$

D. $\square \div \frac{\square}{\square} =$

E. Is it possible to make a generalization about the size of the result obtained when dividing a whole number by a value larger than 1 (e.g., is the result always smaller or larger than the original number)? Explain.

F. Is it possible to make a generalization about the size of the result obtained when dividing a whole number by a value smaller than 1 (e.g., is the result always smaller or larger than the original number)? Explain.

Discussion questions

1. Describe in writing the process your group used to arrive at the solutions. If your group used more than one solution process, comment on the effectiveness of each.

   - Contrast (a.) the "brute force" method (e.g., plugging-in most of the possible combinations of numbers) with any other methods used.

2. Generalize your finding to any sequence of numbers (e.g., 50 through 56).

3. Explain why each solution is correct (by more than merely re-doing the arithmetic).

4. Do you see any relationship between the answers to 2A through 2D? Between 2C and 2D? If your knew the answer to 2C, could you have used what you know about multiplication and division of fractions to have predicted the answer to 2D?

Math 211: Additional Group Work: Attaching meanings to operations

Show your understanding of mixed numbers and the concept of multiplication by illustrating these with pattern blocks:

$$2\frac{1}{3} \times 1\frac{1}{2}$$

$$1\frac{1}{2} \times 2\frac{1}{3}$$

$$1\frac{2}{3} \times 2\frac{1}{2}$$

$$2\frac{1}{4} \times 2\frac{2}{3}$$

$$2\frac{2}{3} \times 2\frac{1}{2}$$

Describe the pattern blocks involved at each step of your work, not just the final arrangement. Give also the numerical value of the block answer.

Let the hexagon = one-half. Show your understanding of the concept of division by describing a pattern block solution of

$$\frac{3}{4} \div \frac{1}{3}$$

Describe the pattern blocks involved at each step of your work, not just the final arrangement. Give also the numerical value of the block answer.

MATH 211 Reflection Assignment, due Mon., 12 Mar

Spend about 20 minutes reflecting. ("Reflect" does not mean "off the top of your head.") Then write your reflection (roughly both sides of this sheet, depending on the size of your writing).

THE SUGGESTIONS ARE NOT AN OUTLINE BUT ARE MERELY AN INDICATION OF THE TYPES OF THINGS THAT MIGHT ARISE DURING A REFLECTION.

- Reflect on the last quiz (take-home part and in-class part). If you did well, to what do you credit your success? Did you write answers quickly with understanding, or did you have to look several things up or even seek help? If you didn't do so well, why did you miss items (e.g., lack of genuine understanding, lack of preparation, general lack of effort,. . .)? What are your thoughts on a grade on a group effort? Etc.

- Reflect on your work, overall, so far in this course. (Your growth in mathematics; your work in the groups, on homework, on take-home quizzes; your attendance; your seeking and finding help when you don't understand something; your quantity of work; your quality of work; are you working like you would hope the teachers of your children worked when they were in college; etc.)

Proportional Reasoning--Lesson 1

MATH 211 Group Activity on ...

1. (Areas in nonstandard units) Find the areas of the shapes on the separate sheet in trapezoids and in blue rhombuses.

Shape A. _____ trapezoids

_____ blue rhombuses

Shape B. _______ trapezoids

_______ rhombuses

(Do these next 3 parts last in case your group runs short of time: Work individually and then compare your thinking.)

Invisible shape X has area 16 trapezoids. What is its area in blue rhombuses? Explain your reasoning.

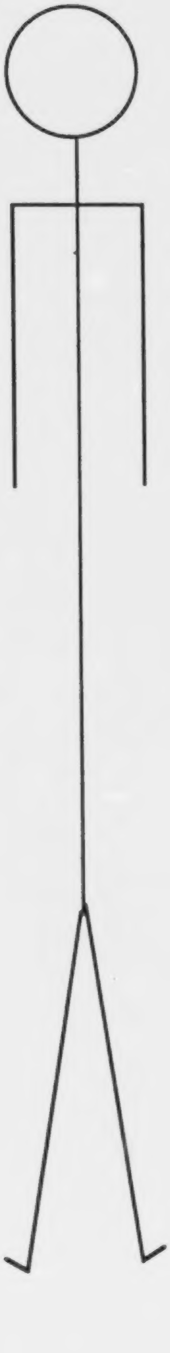
Invisible shape Y has area 21 trapezoids. What is its area in blue rhombuses? Explain your reasoning.

Invisible shape Z has area 24 blue rhombuses. What is its area in trapezoids? Explain your reasoning.

2. (Group work) Each of two boxes has a mixture of red (R) cubes and green (G) cubes. In each line, which of the two boxes gives you the better chance of getting a red cube, if you shake up the boxes and draw without looking?

	Box 1	Box 2	Because...
A.	1 R, 1 G	2 R, 2 G	
B.	2 R, 3 G	1 R, 2 G	
C.	2 R, 3 G	6 R, 7 G	
D.	20 R, 1 G	100 R, 1 G	
E.	5 R, 3 G	4 R, 1 G	

3. (Group work) Mr. Short is 4 large buttons in height. Mr. Tall, who is not pictured here, is similar to Mr. Short but is 6 buttons in height. Measure Mr. Short's height in paper clips and predict the height of Mr. Tall in paper clips. Explain your prediction.

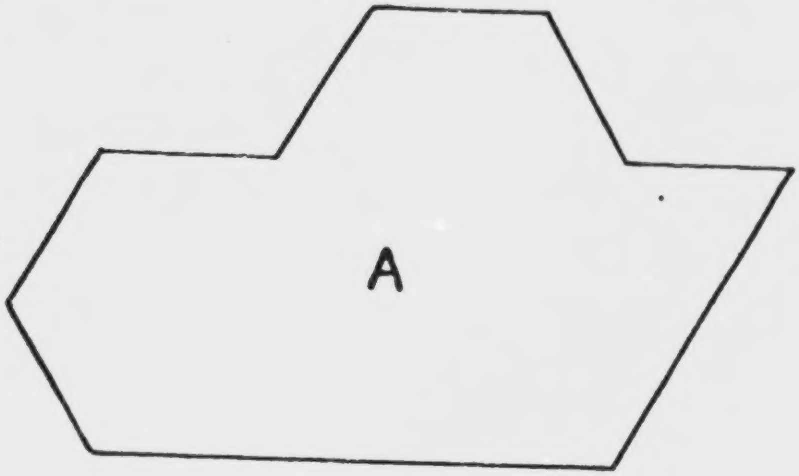


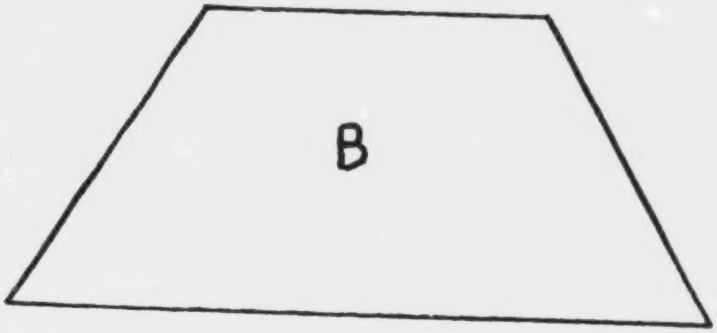
4. (Group work) A paper bag has cubes of three colors in it; there are 12 cubes in all in the bag. Your group is to decide exactly what is in the bag, without looking. All you are allowed to do is this:

Shake the bag, draw one cube without looking, note its color, and then put the cube back into the bag (without looking!).

You may repeat the allowed actions as many times as you wish.

These shapes originally corresponded to the sizes of pattern blocks. They may have been affected by reproduction and so need to be redrawn with pattern blocks before use.





Proportional Reasoning -- Lesson 2

MATH 211 Some of the Ways Two Variables Can Be Related

Here are some (of the many) possible relationships between two quantities x and y in different situations:

$x + y$ always equals the same number ($x + y = \text{constant}$)
$x - y$ always equals the same number ($x - y = \text{constant}$)
xy always equals the same number ($xy = \text{constant}$)
$x \div y$ always equals the same number ($x \div y = \text{constant}$).

Decide how x and y are related in the following situations, by seeing what happens as x changes. (If x and y are not related in any of the ways above, say so.)

x is...	y is...
A. Your mother's age	your age, at different periods of time
B. Your number of points correct	your number of points off, on examinations worth 100.
C. Percent chance of rain	percent chance of no rain
D. The shaded part of a whole	the unshaded part of the same whole
E. Speed driving, SDSU - home	time it takes, SDSU - home
F. Sales tax charged	price of item
G. Length	width, of rectangle with area 36 cm^2
H. Number of identical items you can buy with \$20	price of each item
I. Time expired in a pro basketball game	time left
J. Change from \$20	cost of item bought
K. Length in centimetres	length of same item in inches
L. Value of item in pesos	value of same item in dollars

Math 211: Measures of Central Tendency: Group Work

Use data on age in months (for this class) for the first five questions.

1. Compute the mean (arithmetic average) age of students in this class

2. Find the median (middle age of students in this class
(Hint: first order the date. If there are an odd number of data, find the middle number. If there are an even number of data, take the average of the two middle numbers)

3. Which measure of central tendency, the mean, or the median, is more influenced by outliers?

4. Which measure is more representative of the "average age" of students in class? Why? Which would be less influenced by including Larry's and Judy's ages?

5. Put a line in the ordered data where the median is. Now find the median of the date above the line (or to the left of the line) and the median of the data below the line (or to the right of the line).
Now locate five values that could be considered the "lower extreme", the "lower quartile", the "median", the "upper quartile", and the "upper extreme". Name them.

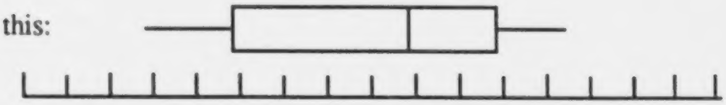
le = lq = med = uq = ue =

6. Next you will make a box-and-whisker graph for age in months, accompanied by a number line.

235

First draw a line segment where one endpoint is the le, the other end point is the ue. Then mark the place where the median is, and where each quartile is. Box in the quartiles.

(It should look something like this:



7. Is the median the mean of the upper and lower quartiles?

Is it always?

Is it always close?

8. The range is ue - le. The interquartile range is uq - lq
Is the interquartile range half the range? Why or why not?

9. Is the median always between the upper and lower quartiles? Is the mean always between these two quartiles? If not, illustrate cases where it is not, by choosing special sets of data.

10. Back to the data: The mode is the most frequent. What is the class mode for age in months? For age in years?

11. If a member is more than 1.5 times the interquartile range above the upper quartile or below the lower quartile, it is formally called an outlier. (We have used this term by "eyeballing" the data up to now.) Are there any outliers in this data?

12. A student came from Grossmont with a grade point average of 2.8 from her 24 semester hours taken there. She has taken six semester hours at SDSU and has a gpa of 3.2. What is her overall gpa?

13. Another student came from Mesa with a gpa of 2 after taking 6 semester hours. At SDSU he has earned a gpa of 3.6 after taking 30 semester hours. What is his overall gpa?

14. The average weight of the females in a class is 129. The average weight of the males is 149. What is the average weight of the students in the class?

ASSIGNMENT:

(1) Read pp. 340-344; 346;359 (except computer programs). Do #1, 3 on p.350.
(2) Make two box plots: one of building heights in SF; one of LA, using the same number line. Make a box plot of the number of hamburgers eaten this month by people in this class, and write and interpretation.
(3) For each of the sets of data we collected, discuss whether or not it is possible to find the mean, the median, or the mode. In cases where you can find more than one, is any one measure better than another? Why or why not?

Probability--Lesson 2

Math 211: Determining the Experimental Probabilities for the Two-Dice Sums Game: Group Work

1. Using the data from your team's play of the Two-Dice Sums game, give experimental probabilities for each of the sums (in both fraction and decimal forms).

P(sum = 2) = or

P(sum = 3) = or

P(sum = 4) = or

P(sum = 5) = or

P(sum = 6) = or

P(sum = 7) = or

P(sum = 8) = or

P(sum = 9) = or

P(sum = 10) = or

P(sum = 11) = or

P(sum = 12) = or

2. Record your team's results on the class chart.

Join another team, and discuss these questions.

3. Were there any surprises in individual team results? How did your results compare with the class results?

4. Which sum occurs most frequently? (Which sum is most probable?)

5. Which sum occurs least frequently? (Which sum is least probable?)

6. Which would be a better bet: an even sum, or an odd sum?

Name ____________________

Two-Dice Sums

Roll 2 dice. Mark an X under the sum. Do this until one sum reaches the Finish Line.

2	3	4	5	6	7	8	9	10	11	12
Finish Line										

(Complete before Monday, 7 May, so you are ready for discussion)

Theoretical Probabilities for Two-Dice Sums

1. Experiment: Toss a red die and a green die, note the number of dots on top of each, record in ordered pair form. (Notice that this experiment does not call for finding the sum.) How many possible outcomes are there? ____ Are they equally likely? ___

2. Now fill in the sums you would get for all the possibilities.

Green Die

Red Die

+	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

3. Finish this table to see how many ways each of the possible sums can happen.

Possible sum =	1	2	3	4	5	6	7	8	9	10	11	12
				2,2 3,1 1,3								
Total * of ways				3								

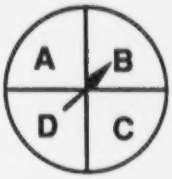
4. Using the table in #3, give the theoretical probability for each possible sum.

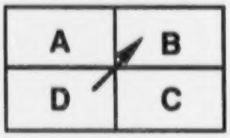
5. Compare these theoretical probabilities to the experimental probabilities obtained earlier.

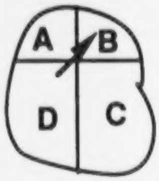
Probability--Lesson 6

MATH 211 Concept Questions on Probability
Have ready to hand in/discuss Friday, 11 May.

1. Consider the following spinners. Which one or ones give you the best chances of getting a C when you spin (or are the chances the same)? Justify your answer.







2. In a certain lottery, the probability of winning at least a few dollars (and perhaps more) is 1/10. You have bought 9 lottery tickets, and none of them won anything. What is the probability that the next ticket you buy will win something?
Discuss.

3. A. Game 1. You use just the 2 through 12 part of a number line. You have 11 markers. You can put them anywhere you like on the 11 numbers 2 through 12, repeating a number if you wish. The teacher tosses two regular dice. If you have a marker on the sum for the toss, you can remove the marker. The object is to be the first one to remove all 11 markers.

Where would you put your markers to give you a good chance of winning? Why?

B. Game 2 is like Game 1 except the teacher is using just one die, you use only 1 through 6 on the number line, and you have 6 markers. The object is still to be the first one to remove all the markers. Where would you put your markers to win, and why?

Some Probability Problems

MATH 211

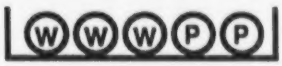
Have ready for Wednesday, 16 May.

1. Suppose that there are 3 essential components on a space shuttle launch: rocket system, guidance system, and communication system. Suppose also that the probabilities that the systems work are these:

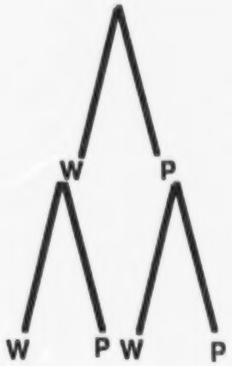
rocket	0.95
guidance	0.9
communications	0.99

If one or more of the components do not work, the launch must be aborted. What is the probability that a launch will be aborted? [We are not taking into account back-up systems.]

2. Experiment:
   1. First, randomly draw a ball from the box and note its color.
   2. Then replace the ball and repeat Step 1.



Complete this probability tree diagram:



A. Find the probability of each outcome.
B. What is the probability that you get the same color on both draws?

3. Repeat #2, except that the ball is not replaced (drawing without replacement).

APPENDIX B.4

MATH 211
FINAL EXAMINATION

Final Exam

Math 211 Spring 1990
Name______________________________

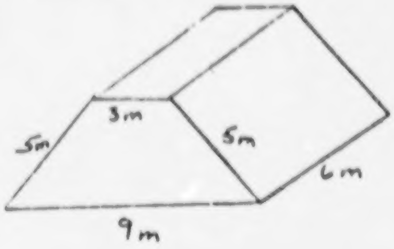
Point values are in the margin; the total is 187, to be weighed 30% of your grade.

(3+3+2) 1. The building is 4 metres high. What are the following? (Give units also.)

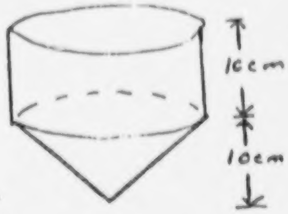
a) its surface area (not counting the floor)_________

b) its volume_______________

c) the perimeter of the floor____________



(4) 2. What part of the capacity, or volume, of the whole funnel to the right is in the bottom piece? ________



(6) 3. "Think metric" to fill the blanks.

a) The child is 1.2 _________ tall. b) 4 cm^2 = __________ dm^2 c) 7.34 kg =_________ g

(8) 4. Place the following in order, from smallest to largest:

$1\frac{3}{5}$, 95%, $\frac{15}{12}$, 145%, $1\frac{14}{21}$, $1\frac{14}{22}$, 1.82 %, $\frac{1}{17}$

(4) 5. Show, using the O's below, that $\frac{2}{10} = \frac{4}{20}$

O O O O O
O O O O O
O O O O O
O O O O O

(6) 6. Show how you would use the benchmark method. . .

a) to find a fraction between $\frac{8}{15}$ and $\frac{14}{29}$

b) to choose the larger of $\frac{12}{39}$ and $\frac{7}{18}$

(6) 7. Tell how you would find estimates for the following:

a) 97% of 17.85

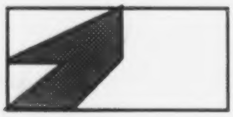
b) $5.1 \div 0.33$

(6) 8. Explain or show how the following could be done mentally.

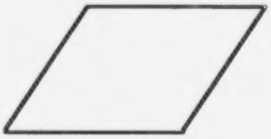
a) 75% of 480

b) $23 - 17\frac{3}{8}$

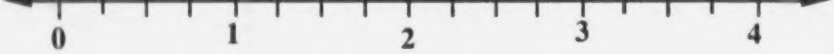
(6) 9. a) What fraction of the region is shaded? _______



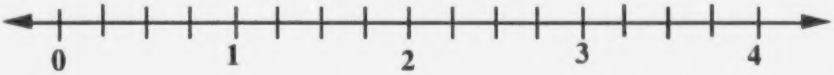
b) This is $\frac{3}{5}$ of a whole. Show the whole.



(7) 10. a) Show how to find $\frac{1}{2} \times \frac{3}{4}$ on the number line.



b) Show how to find $2\frac{1}{4} \div \frac{1}{2}$ using a number line.



(21) 11. Write as fractions (in $\frac{\text{whole number}}{\text{whole number}}$) form. If it is not possible, say so, and tell why.

a) 2.1999

b) 2.1999. . . .

c) 2.191991999...

d) 2.19%

e) $\sqrt{2}$

f) $\frac{2\sqrt{2}}{3\sqrt{2}}$

g) $\frac{2/3}{0}$

(8) 12. Continue the story problem that is started, so that the problem fits the indicated calculation. (You do not have to give the answer.)

a) ($6\frac{1}{2}$ - $4\frac{3}{4}$ comparison) Carolyn bought $6\frac{1}{2}$ pounds of apples...

b) ($4.5 \div 0.6$ repeated subtraction) A pharmacist has 4.5 ml of a certain medication...

(4) 13. Write a story problem to show that $3 \div 8 = \frac{3}{8}$
(You may also use a diagram if you wish.)

(4) 14. Make a drawing to show $3 \times \frac{3}{4}$.

(4) 15. Sketch the solution of 1.2 - 1.03 with place value materials, using the missing addend viewpoint.
(Use □ | x for place-value materials.)

(6) 16. a) Sketch pattern block work that shows how many $\frac{2}{3}$s are in $2\frac{1}{3}$. Tell what your unit is.

b) Write an equation for your work in part (a). ____________________

(4) 17. Which box gives you a better chance of drawing a **B**lack ball? Explain.

Box **X**	Box **Y**
Ⓑ Ⓑ Ⓑ Ⓦ Ⓦ	Ⓑ Ⓑ Ⓑ Ⓑ Ⓦ Ⓦ Ⓦ

(26) 18. True or false: Circle T if always true, F if not always true.

T F (a) A student who gets 3 out of 5 questions right on an exam scores at the 60th percentile.

T F (b) A z-score of 3.41 on an SAT indicates that the student will probably be in honors classes in a University.

T F (c) A z-score of - 0.67 on a class exam indicates that a student may be in danger of failing that class.

T F (d) For a given exam, the greater the standard deviation, the greater the chance of outliers.

T F (e) A student has a 3.2 GPA from classes taken at SDSU, and a 2.8 GPA from classes taken at Grossmont Community College. The overall GPA of the student is 3.

T F (f) 75% of the scores on a test are rarely above the lower quartile.

T F (g) For a normal distribution, the mean, median and mode all have the same value.

T F (h) One feature of Logo is that one can "teach" it new words.

T F (i) SI is used by only a few people in the world, except for scientists.

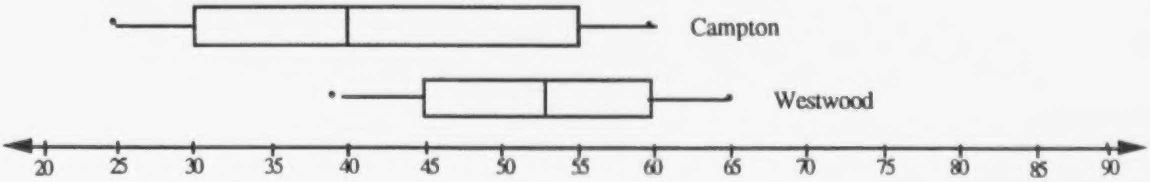
T F (j) Pascal's triangle can be used in any multi-step probability experiment.

T F (k) Tossing a fair coin 80 times could result in 36 heads.

T F (l) After 5 straight heads on tosses of a fair coin, the probability of heads on a sixth toss would be $\frac{1}{2}$.

T F (m) If an experiment has 4 outcomes, the probability of each outcome is $\frac{1}{4}$.

(5) 19. The ages of teachers in Campton and Westwood school districts are shown in the box-and-whisker graphs below.



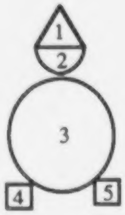
Write an interpretation of these graphs, as though you were presenting these data to the two superintendents.

(5) 20. In a science class of 17 students you learn that on the last test, you received an 82 and the class mean was 75. In a math class of 21 students you learn that on the last test you received an 82 and the class median was 75. In which class do you think you did better on the exam, and why?

(5) 21. In 6 tosses of a fair coin, what is the probability of getting heads exactly 2 times?

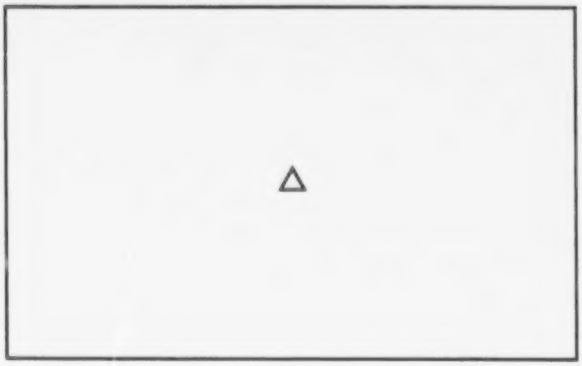
(4) 22. An insurance company decides that the probability of someone your age and sex having a car accident in a year is a certain number. Is that probability an experimental or a theoretical probability? Explain your answer.

(4) 23. There are 6 colors of crayons. In how many visually different ways can you color the figure to the right? You may use the same color more than once, but each numbered region is all one color.

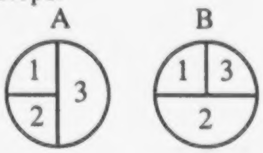


(5) 24. Show how the screen would look after this set of Logo commands:

```
DRAW
REPEAT 2[LT 90 FD 30]
RT 90
BK 60
HOME
```



(8) 25. Experiment: Spin each spinner, and notice where the spinners stops.
(If the spinner ends on a line, spin again.)



a) Make a probability tree diagram for the experiment.

b) Find the probability that the <u>sum</u> of the two numbers is 4.

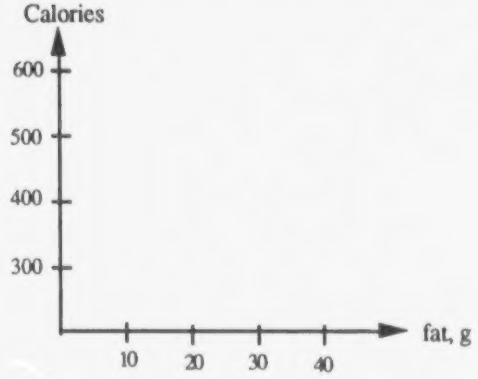
(9) 26. This table gives the grams of fat and the number of calories in some popular fast food items.

Food item	g of fat	Calories
Burger King Whopper	41	660
Jack-in-the-Box Jumbo Jack	28	538
MacDonald's Big Mac	33	591
Wendy's Old Fashioned	22	413
Roy Rogers Roast Beef	12	356
Hardie's Roast Beef	17	351
Arby's Roast Beef	15	370
Long John Silver's Fish	27	483
MacDonald's Filet-O-Fish	18	383
Kentucky Fried Chicken Snack Box	21	405

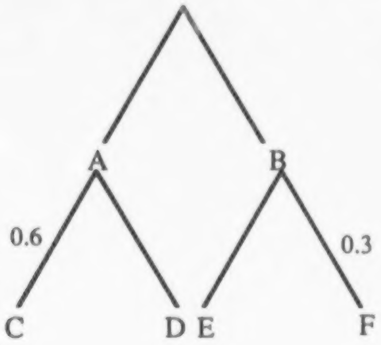
a) Make a stem-and-leaf plot of the grams of fat, and write an interpretation. (Data from Sept. 79)

Interpretation:

b) Below is a scatter plot of the data above. Sketch the regression line and write a sentence describing what it says to you.



(4) 27. To the right is an incomplete probability tree diagram for an experiment. If the probability of AC is 0.12, what is the probability of BE?________



End of examination. Good luck in MATH 312/309 and in your teaching career.

APPENDIX B.5

MATH 211
QUESTIONNAIRE
ON
LEARNER GOALS/THEORIES OF INTELLIGENCE

Responses to Open-ended Items on Questionnaire
(Respondents Coded. Experimental and KT's classes)

When you feel smart in math, what makes you feel smart?

SA: That I can do it--accomplishment! Also, very importantly the teacher - of a math course makes me feel smart and confident - if they are patient and helpful in my abilities. Usually it comes 2nd nature to them and they don't put in the effort. I get very discouraged when I don't get good direction.

DC: When I find the right way to solve a type of problem.

GI: When I try doing a problem and get it right--especially when I know I've figured out what I done wrong before and being able to correct the mistake on my own or w/a little hint.

KD: Helping others who have trouble understanding something I understand.

ND: When I am able to work out a difficult problem and get the correct answer.

TE: When I accomplish something I didn't feel possible.

NE: When I understand the process and can think it through.

TF: That I could use my thinking processes to get an answer to a difficult problem.

MF: Ability to come up w/ correct answer after hard work of trying to understand it.

QF: When I'm able to figure the answer out on my own and have it be correct.

EG: Getting a good grade on tests. Able to solve most answers given to me.

IH: Simply, that I arrived at a right (correct) answer.

BH: Getting the answers right after struggling with a problem.

SI: Being able to finish problems from start to finish correctly.

MM: Studying for a test or doing homework and getting the concept of the problems right. Or taking a test and performing well.

TN: When I have attempted hard problems or new ones and do them correctly. I also enjoy learning why you work things in certain ways.

DN: When I see that I can solve a hard problem.

LN: When I do well on exams, but also when I really understand the procedure in attaining a particular answer.

NN1: The fact that I did well on a math test. My overall grade in the class.

NN2: When I understand what I am doing or when I overcome something that has been troubling me.

KN: Understanding what is being taught and being able to explain it to others would be when I feel smart.

MU: The correct answer.

DU: Conquering a problem presented to me.

UX: When I know how to approach the problem and get the correct answer.

1: When I look at a problem at first and it seems incredibly hard and impossible then I work it out and break it down and understand it and get the right answer.

2: Watching myself work through a problem and come up w/ a solution.

3: That I can understand how to start a problem.

4: Problem solving.

5: Trying to solve a problem--working on it and then finding a solution.

6: The fact that I can do now what I couldn't in the past.

7: When I follow the appropriate steps and end up with a correct answer.

8: Doing all the necessary steps to reach a conclusion and getting it right.

9: A good test grade.

10: Figuring out a hard problem and then explaining the solution to others.

11: When I understand why and how I got a problem right.

12: Getting a good math test score.

13: After trial and error, I come upon the answer completely on my own.

14: I feel smart when I understand it.

15: When I come to the conclusion, or reach the answer by my own doings and thinking.

16: Understanding the problem.
17: Being able to find the solution myself, even if it takes me longer than the next person.
18: When I understand the problem and get the answer correct.
19: To feel sure that I am capable of completing slightly complicated daily problems.
20: Getting an answer correct or when getting a problem wrong, shown the correct way and using that new knowledge to get right later.
21: Working at something challenging and being able to solve it.
22: Figuring out the computations on my own.
23: When I correctly find a solution to a challenging and stimulating problem.
24: When I have a hard problem and I keep working on it all by myself, even if a try to do the wrong way and have to start all over, when I finally get the right answer I am proud of myself for thinking for myself.
25: Understanding the concept!
1a: When I can accomplish the task using the knowledge that I have.
2a: When there are concepts I can understand and apply other math problems to.
3a: Getting the right answer. Sometimes just grasping the concept!
4a: I feel smart when I can figure out word problems.
5a: When you do well on a math test after understanding the homework.
6a: The ability to grasp a new concept.
7a: Solving problems that at one point I had a hard time with.
8a: If I feel comfortable with the problem and I don't feel anxiety just reading it.
9a: That I understand the concepts and study hard.
10a: Understanding the concepts well enough to do given problems.
11a: When I catch on right away. When I really understand what is happening.
12a: The challenge, to know you struggled and still figured the problem makes you feel good.
13a: When I figure something out when I thought I never could or would.
14a: When I fully understand it, and all my answers come out correct.
15a: Being able to understand the reason and process behind the problem.
16a: Using deductive reasoning.
17a: That I was able to apply a concept or formula I learned in class to the problem.
18a: When I've done a problem right that I couldn't do before.
19a: 1. When I accomplish a high score on an exam. 2. When I work a problem out and it's correct.
20a: When I understand what I'm doing.
21a: How much thought I put into it to get the right answer.
22a: When I come to a conceptual answer that is parallel to the numerical one.
23a: When I grasp a concept that is difficult.
24a: That I was able to do a hard problem on my own and not ask anyone for help.
25a: Understanding the problem.
26a: I like word problems, they are more fun than equations--algebra is what I'm best at, so I feel semi-smart in that.
27a: The fact that I spent a good deal of time working out the different approaches to the problem. Putting effort even if it might not be right.
28a: Knowing I did it myself without time pressures.

When you feel stupid in math, what makes you feel stupid?

SA: When everyone is finished with a math problem but I haven't even started. And when they raise their hands to ask excellent questions--that I don't know.
DC: When I can't solve a problem after I have tried over and over. *I don't feel stupid, only frustrated.
GI: 1) When I missed a "little" mistake. 2) When I'm completely stumped by a new lesson.
KD: Usually simple mistakes when solving problems which lead to a wrong answer.
ND: When I make a stupid mistake--I miss something that I knew because I was careless.

TE: When others put me down for making a wrong answer.
NE: All of the stupid (+) and (-) and small errors that I make and FRACTIONS screw me up.
TF: That sometimes I can not "get" how to do a problem in one explanation.
MF: Inability to grasp a concept after much explanation.
QF: When, no matter what I try, I can never come up with the correct answer.
EG: Not being able to solve problems when most understand, but I do not.
IH: When everybody in the classroom gets the concept right away and I don't.
BH: Being at a loss when trying to solve a problem.
SI: I feel down when I haven't prepared properly for a test and it shows.
MM: Seeing a problem I've done over and over and I forget how to do it. Or, do a small math error that shouldn't have been made.
TN: I do not feel I can understand many of the harder problems or courses so when I see my boyfriend miss an "easy" problem in calculus it makes me frustrated.
DN: When I don't have any idea on how to solve a problem.
LN: When I make careless (super careless ones) mistakes.
NN1: When I make stupid mistakes, such as adding two numbers together and getting the wrong answer.
NN2: When things seem to be very easy to everyone else and I just don't seem to get or only partly get it.
KN: I feel stupid in math when I don't grasp the concept of what is being explained or taught.
MU: Being stumped, not getting the answer when Suzy Q and the rest of the class understand the concept and application.
DU: Frustration, usually because I missed a step.
UX: When I have no idea of how to begin the problem.
1: When I work hard at a problem and try and try and I still can't get it.
2: When I follow a wrong path at finding the solution.
3: When I don't even have an idea where the problem is heading.
4: Stupid mistakes.
5: If I give up w/o trying.
6: The fact that I cannot do it, or learn it.
7: When I continually fail to understand one unit. I may do extremely well on one unit and terrible on the next.
8: When I look at a problem forever and still don't know how to approach it.
9: A low test score.
10: Not being able to complete a problem--word problems often make me feel stupid.
11: When I feel lost and don't understand where answers are coming from.
12: Getting a bad math test score.
13: When the simplest things stump me when I'm on my own, but are so obvious when someone explains it to me. Also, when I study really hard, but still do badly on the test because of nervousness, going blank, or careless mistakes.
14: When others all understand and I don't.
15: When I rush through simple things that I know I'm capable of but have carelessly worked on it.
16: When I don't take the time to study.
17: When I struggle for a long time and still can't see the answer, and then someone just runs through it and comes up w/ the correct answer, and I still don't see how they solved it.
18: When I make dumb nervous computation math mistakes.
19: Not knowing what is going on in class and not being able to participate because of my insecurity towards math.
20: Word problems, probability and odds make me feel stupid because I'm supposed to be teaching this to K-6 and I can't get it. Especially when I have a problem and am told it came out of an elementary school text.
21: I feel stupid when I let a problem get to me and make me so mad that I give up.

22: When something is very obvious and I make a stupid mistake (like adding or subtracting something wrong in one of the steps).
23: When I do simple "stupid" mistakes to problems that I know how to do. I feel stupid if I misread or overlook an element in the problem.
24: What makes me feel stupid is not being able to even start a problem, because I don't know what it wants. This usually happens with word problems!
25: Making dumb mistakes, especially when obvious.
1a: When I make mistakes that I could have avoided if I would have been paying more attention to what I was doing.
2a: If I try very hard on a problem and still can't figure it out, then, in class for example, I find an easy solution to it.
3a: I feel stupid when the concept has been explained, everyone around understand, and I'm still in a fog!
4a: I feel stupid when I make careless mistakes.
5a: Failing or getting worst grade in class.
6a: Coming up with the wrong, only because I was rushed and didn't take the time to think.
7a: Not seeing the obvious. Being one of the few people unable to solve a problem.
8a: When someone is trying to explain it to me and it sounds like a different language.
9a: It's only when I don't study at all and do poorly or when I do study and then fail or do poorly because of test anxiety--actually, this doesn't really make me feel "stupid," just frustrated.
10a: Not grasping the concepts. I usually do well in math but the one thing I never really grasped was geometry. However, I breezed through trig.
11a: Not having a clue to what is going on. Then I need to go back and work through it.
12a: Stupid mistakes, when I know better.
13a: When I have a mental block and forget easy equations.
14a: When I get frustrated at not being able to grasp concepts--which is too frequent.
15a: That I don't understand or cannot grasp the principal idea. The teacher plays a part. Some teachers (here) when you ask them a question, they answer and when you still do not understand, they get mad.
16a: Letting frustration get the best of me.
17a: Hard problems.
18a: When I can't even start a problem.
19a: 1. Wrong answers. 2. Not knowing how to do it.
20a: Getting low scores on tests when I thought that I did well.
21a: When it is an easy problem and I find it somewhat difficult.
22a: I don't really feel stupid in math. I feel frustrated when I can't see a simple problem solving process.
23a: When I can't comprehend something that I know is probably quite simple.
24a: Nothing really makes me feel stupid except when someone who got a higher score than me flaunts their score.
25a: Not understanding the problem, especially when most others do.
26a: Some formulas are so complex, I can't memorize them very well, and if I do I don't know what it means--so I feel stupid sometimes.
27a: Careless mistakes like going too fast through a problem and not taking the time to work it out w/ patience.
28a: When I'm rushed and I'm not allowed to give it my best.

Results for Experimental MATH 210B Class (n=25) [first entries],
and Experimental Class Plus Second 210B Class (n=53) [second entries]

Name Results

There are no "right" or "wrong" answers on this survey. The only correct responses are those that are true for you. Do not spend much time with any statement, but be sure to answer every statement. Work fast but carefully. Respond with the appropriate letter(s) in the blanks:

A = agree SA = slightly agree U = undecided
SD = slightly disagree D = disagree

___ 1. I am sure of myself when I do math.
A. 20% SA. 56% U. 12% SD. 12% D. 0
A. 22.6% SA. 39.6% U. 11.3% SD. 18.9% D. 7.6%

___ 2. Intelligence is something you can increase if you want to.
A. 84% SA. 4% U. 4% SD. 4% D. 4%
A. 71.7% SA. 13.2% U. 5.7% SD. 1.9% D. 7.6%

___ 3. Good math students almost never get problems wrong.
A. 4% SA. 16% U. 4% SD. 32% D. 44%
A. 5.7% SA. 11.3% U. 3.8% SD. 24.5% D. 54.7%

___ 4. You can learn new things, but how intelligent you are stays pretty much the same.
A. 4% SA. 20% U. 20% SD. 12% D. 44%
A. 9.4% SA. 20.8% U. 18.9% SD. 15.1% D. 35.8%

___ 5. I don't understand how people can enjoy spending a lot of time on math.
A. 4% SA. 12% U. 8% SD. 44% D. 28%
A. 11.5% SA. 11.5% U. 11.5% SD. 30.8% D. 34.6% [n = 52]

___ 6. If a person is not good in math, it doesn't matter how hard he/she works.
A. 0 SA. 0 U. 0 SD. 20% D. 80%
A. 1.9% SA. 1.9% U. 1.9% SD. 18.9% D. 75.5%

___ 7. Once I start trying to work on a math puzzle, I find it hard to stop.
A. 24% SA. 28% U. 24% SD. 12% D. 12%
A. 22.6T SA. 22.6% U. 15.1% SD. 17% D. 22.6%

___ 8. When I do well in math, I ask myself why I did so well.
A. 20% SA. 16% U. 8% SD. 36% D. 16%
A. 17.3% SA. 21.2% U. 11.5% SD. 21.2% D. 28.8% [n = 52]

___ 9. I'm not the type to do well in mathematics.
A. 12% SA. 12% U. 8% SD. 20% D. 48%
A. 20.8% SA. 17% U. 7.6% SD. 18.9% D. 35.8%

___10. Males are by nature better at math than females are.
A. 0 SA. 8% U. 8% SD. 12% D. 72%
A. 1.9% SA. 5.7% U. 7.6% SD. 15.1% D. 69.8%

___11. You can change how intelligent you are in math.
A. 56% SA. 24% U. 16% SD. 0 D. 4%
A. 54.7% SA. 24.5% U. 11.3% SD. 3.8% D. 5.7%

___12. I don't think I could do well in a calculus level course.
A. 24% SA. 20% U. 24% SD. 16% D. 16%
A. 28.3% SA. 18.9% U. 24.5% SD. 13.2% D. 15.1%

___13. Math is fun and exciting.
A. 16% SA. 48% U. 12% SD. 20% D. 4%
A. 13.2% SA. 34% U. 11.3% SD. 22.6% D. 18.9%

A = agree SA = slightly agree U = undecided
SD = slightly disagree D = disagree

___14. I prefer hard, new, and different tasks so I can try to learn from them.
A. 24% SA. 48% U. 4% SD. 12% D. 12%
A. 22.6% SA. 37.7% U. 5.7% SD. 20.8% D. 13.2%
___15. Making a mistake shows you are dumb in math.
A. 0 SA. 0 U. 0 SD. 8% D. 92%
A. 0 SA. 0 U. 0 SD. 5.7% D. 94.3%
___16. I'm not good in math.
A. 8% SA. 8% U. 16% SD. 20% D. 48%
A. 13.2% SA. 17% U. 13.2% SD. 22.6% D. 34%
___17. I like things I'm good at so I can feel intelligent.
A. 36% SA. 48% U. 4% SD. 12% D. 0
A. 39.5% SA. 41.5% U. 3.8% SD. 11.3% D. 3.8%
___18. Mathematics problems always have just one right answer.
A. 16% SA. 16% U. 8% SD. 16% D. 48%
A. 11.3% SA. 13.2% U. 5.7% SD. 24.5% D. 45.3%
___19. I prefer tasks that are fun and easy to do, so I don't have to worry about mistakes.
A. 12% SA. 8% U. 4% SD. 52% D. 24%
A. 7.6% SA. 20.8% U. 9.4% SD. 37.7% D. 24.5%
___20. Mathematics is as important for females as for males.
A. 96% SA. 0 U. 0 SD. 0 D. 4%
A. 98.1% SA. 0 U. 0 SD. 0 D. 1.9%
___21. It's always better to use a calculator for calculations if one is available.
A. 20% SA. 24% U. 12% SD. 12% D. 32%
A. 30.2% SA. 18.9% U. 9.4% SD. 15.1% D. 26.4%
___22. I like problems that are hard enough to show others that I'm intelligent.
A. 12% SA. 12% U. 28% SD. 20% D. 28%
A. 11.3% SA. 13.2% U. 24.5% SD. 17% D. 34%
___23. If I do poorly on a math test, I ask myself why I didn't do better.
A. 76% SA. 16% U. 0 SD. 4% D. 4%
A. 71.7% SA. 15.1% U. 1.9% SD. 3.8% D. 3.8%
___24. There is always a best way to do things in mathematics.
A. 16% SA. 12% U. 8% SD. 28% D. 36%
A. 13.2% SA. 15.1% U. 13.2% SD. 26.4% D. 32.1%
___25. Hard work can increase my ability to do math.
A. 72% SA. 20% U. 4% SD. 0 D. 4%
A. 69.8% SA. 18.9% U. 1.9% SD. 5.7% D. 3.8%
___26. I like problems that aren't too hard, so I don't get many wrong.
A. 24% SA. 28% U. 12% SD. 20% D. 16%
A. 20.8% SA. 30.2% U. 11.3% SD. 20.8% D. 17%
___27. If I can't solve a math problem right away, I stick with it until I do.
A. 28% SA. 48% U. 4% SD. 8% D. 12%
A. 22.6% SA. 43.4% U. 5.7% SD. 17% D. 11.3%
___28. I would rather have someone give me the solution to a hard math problem than to work it out for myself.
A. 8% SA. 4% U. 4% SD. 24% D. 60%
A. 9.6% SA. 11.5% U. 5.8% SD. 28.8% D. 44.2% [n = 52]

When you feel smart in math, what makes you feel smart?
When you feel stupid in math, what makes you feel stupid?
Check to see that you have responded to every item, and then turn your survey sheet upside-down.

APPENDIX B.6

MATH 211
FINAL QUESTIONNAIRE
ON
LEARNER ATTITUDES

Final Questionnaire

Please reflect on each of the following questions and write a brief paragraph on each. You do not need to give your name.

1. What one or two topics do you feel you made most progress on, and why?
2. What topics or concepts do you still feel uncomfortable with? Can you explain why?
3. Many people believe that doing mathematics is basically learning and following a set of rules. Do you agree or disagree? Why?
4. How did your group work help you and/or hinder you in learning?
5. Do you feel that you participated fully in group work? Did others in your group participate fully?
6. Did you make any special efforts to try to make sense of the mathematics in the take-home quizzes? If so, how?
7. How frequently did you ask yourself if your understanding was adequate for a teacher of this mathematics?

Number of hours you studied per week for this class _____
Number of hours you work per week ______
Did you attend class regularly? _____ What grade are you expecting? _____
Highest grade received in a high school math class ______
Highest level math class taken in high school _________________________

Responses to Questionnaire

1. What one or two topics do you feel you made most progress on, and why?

1. Fractions and probabilities. I had had these, but my understanding was too broad.
2. Better understanding of fractions, knowing how to estimate the value before calculating the problem.
3. Estimation of fractions.
4. Explaining what 1/2 x 2/3 is. Better understanding of fractions, and with that I understood percentage also. This class was an excellent review. I thought I knew a lot of mathematics, but as it turned out I was quite rusty. This class REALLY HELPED.
5. Fractions and number sense. These kinds of problems caused me great anxiety in the past. I feel much more math confident.
6. Benchmark method and estimation, basically number sense, because we spent a lot of time on this.
7. Fractions, statistics, and probability.
8. Putting numbers in sequential order.
9. Fractions, probability.
10. Fractions and how they relate to one another as well as how they change when using functions (+, -, x,).
11. Estimation of whole numbers, decimals, fractions. In addition, substration (sic) and division.
12. Fractions: making estimation, and showing explanations for times, and division of fractions.
13. Probability and proportions. I can estimate better and have a better knowledge in number sense, especially with percentages.
14. Statistics and probability because I have not had them before.
15. Number sense and estimation.
16. Probability and statistics.
17. Benchmarks. I always used to have trouble trying to figure out which was larger. Now I feel good.
18. (no answer given)
19. Fractions. It has been something I've never felt terribly comfortable with but I've learned to look at them and use common sense (instead of rules).
20. Fractions.

2. What topics or concepts do you still feel uncomfortable with? Can you explain why?

1. Probability, but I belief (sic) I can do better now than before.
2. Geometry ... I tend to look for formulas.
3. Statistics, interpreting graphs.
4. Still confused on probability. As you said, "sometimes the correct probability doesn't always 'look' correct." Also the wording can mean so much.
5. Statistics and probability.
6. Stats and probability. I felt like we were "pushed" through these topics.
7. Illustrations of fraction division; interpretations of z-scores and deviations.
8. Pascual's (sic) Triangle. I feel like we tried to cram it in ...
9. I can't say right now, I'd have to wait after the final exam to be sure.
10. When we worked on fractions I had it under control--but when you ask me now what 1/2 of 35 is I'd have to reach for an old stand-by routine and not my "math sense."
11. Word problems.
12. Probabilities.
13. I still feel slightly uncomfortable with the difference between 5 x 1/4 and 1/4 x 5.

14. None really.
15. Ratios and averages.
16. Area
17. Deviations.
18. Probabilities.
19. Probability. I have fun with the problems and can easily understand them when you do over them step by step. But when I look at the problem on my own I don't know where to begin.
20. Standard deviation and z-scores.

3. Many people believe that doing mathematics is basically learning and following a set of rules. Do you agree or disagree?

1. One does learn and follow rules in mathematics but there are many other ways to figure out problems. One also needs to understand.
2. At the beginning of the class I would have agreed with this, but now I know that math is much more common sense.
3. I disagree. Not all math problems follow a set of rules ... may be able to solve problems intuitively and without a set of rules.
4. DISAGREE. Rules are good, they allow you to do certain procedures, but the important part is understanding when, where, and why you use certain rules.
5. Disagree. Math is a way of thinking. Up until this point
6. Disagree. You will remember math better if you don't just memorize rules.
7. I disagree. If one can use their number sense and estimation abilities one can do mathematics.
8. Disagree. If you learn by just following the rules than (sic) you don't understand why it works.
9. NO. I now have some feeling of what it is to have MATH SENSE. I've become a MATH FAN without even knowing it.
10. At my point in my education it is hard not to agree because my primary technique is to follow rules. However, new teaching methods (show) me that using "math sense" can really work.
11. Math is not just following rules, you have to understand. When I was in high school I did math in the way that people believe, but now that I am in University and especially in this class I noticed that mathematics involves ideas, primarily understanding of what you are doing.
12. I disagree, but it is still hard for me to get away from my old ways ... following the rules.
13. I agree that there are many rules to be followed, but a good sense of numbers and their value allow for a great deal of "shortcuts."
14. For some people, that is the easiest way to learn math. For me, math has always come easily so I am not sure how to answer.
15. Knowing the rules is important, but they are not the absolute.
16. I disagree. There are many ways to do problems. There isn't always one special rule that will guarantee a correct answer.
17. I still basically agree with that. I still do not feel I am good with math but what I know well are formulas.
18. We were taught to follow rules from day one, but this class has taught me to use math sense and to apply those concepts when doing math.
19. To some extent I agree. But if I got anything from this course it is that number sense is just as useful if not more useful than a set of rules.
20. Disagree. If you can think about why you are doing what you are doing then the whole problem will make a lot more sense.

4. How did group work help and/or hinder you in learning?

1. Helped me the most when working with fractions with pattern blocks.
2. Helpful, you can learn two ways, being taught and teaching.
3. Opened my mind to other ideas.
4. I became stuck in a routine in doing a procedure a certain way. I would only see one alternative and would continue doing that problem the same way over and over. The group work helped me see the many different and easier ways. Also, it's nice to be in a class where you know people.
5. It's really great to have four people explaining a problem. If one way of explaining it didn't make sense, then one of the others usually did.
6. The group work helped me.
7. The group helped me a lot. I was able to see different ways of figuring out problems as well as getting experience explaining concepts.
8. It helped.
9. It was cool.
10. I think the group work was a success. It was a good test for me to see how well I know the material.
11. It helped me learn different strategies.
12. I didn't like groups as much as I thought I would. Some people took control and left others out of the discussion. We really didn't know who was right or wrong when there were many ideas floating around.
13. It helped me because it allowed me to see the way other people came up with answers.
14. It helped. I think in many cases it is easier to get help from you peers. Also you can see how people think.
15. My first group did well because we were at the same level of understanding.
16. It helped. I could find out how other people did the problems. It hindered me in that some people never showed or didn't contribute to the group at all.
17. I loved group work. I got to explain my reasoning even if I was wrong.
18. If I didn't understand something right away I could ask someone in the group for assistance instead of waiting.
19. It definitely helped me. I got a better understanding for the problems when they were talked about. It widened my range of thinking.
20. Very helpful.

5. Do you feel that you participated fully in group work? Did others in your group participate fully?

1. Others participated more than me but I did understand the work.
2. Yes.
3. Yes. Second group I was in seemed quieter, passive.
4. Yes, I did. I'm outgoing and like to talk and give my input whereas others don't feel so comfortable doing that.
5. Yes, but two in my last group never seemed to know what was going on and this was not helpful.
6. Yes.
7. Yes. In my first group one of the members was very passful (sic) but overall everyone participated.
8. Yes, yes.
9. Dominant personalities are always a problem rather than an asset.
10. Yes.
11. Yes.
12. I didn't participate fully all the time because if there was something I didn't understand I held back.
13. Yes. We should have spent more time (as a class) going over assignments.

14. Yes.
15. Yes.
16. Yes, but others did not. I tried to do all of my group work because other were relying on me.
17. Yes. I did, most did.
18. As much as I could. The others participated equally.
19. I put the most into my group that I possibly could. Not everyone fully participated.
20. Yes.

6. Did you make any special effort to try to make sense of the mathematics in the take-home quizzes? If so, how?

1. (no answer given)
2. I'm sure I didn't try hard enough, but I tried to make sense of the work.
3. No.
4. YES! By doing this I learned a few things along the way, a lot more than taking a quiz in class.
5. Take-home quizzes helped me much. I felt like I learned the information better and studied it much more effectively than otherwise.
6. Yes, I sure did. For example, I tried to use visual aids, drawings, cut-outs, etc.
7. I carefully worked through each problem often substituting my own numbers to test my knowledge of the concept.
8. I don't understand the question.
9. All the time. Wholistically.
10. Of course! The quizzes I always worked hard at--when I remembered to do them!
11. Sometimes, especially in those questions where I didn't have a clue. Sometimes I feel I'm building a puzzle to get the idea.
12. As best I could.
13. Yes, but it was difficult yet challenging to calculate answers by myself. The book aided somewhat but some of the questions I feel did not pertain directly to the lecture which made them more challenging and at times frusterating (sic).
14. Yes, but using my notes and book for assistance, and also by checking my answers when the test was returned and correcting my answers and asking questions.
15. (no answer given)
16. I didn't really because I was worried about doing well on the quiz. I liked it though because I didn't feel quite as pressured.
17. Yes. I took extra time. I do not feel I did well though.
18. I tried as best I could to understand and make sense of the quizzes.
19. (no answer given)
20. No.

7. How frequently did you ask yourself if your understanding was adequate for a teacher of mathematics?

1. I belief (sic) I have gained enough knowledge to explain problems better. Before I did not belief (sic) I could sit down and convenience a person that 2 + 2 = 4.
2. I ask myself this question often, when I'm not living up to my capabilities.
3. During the stats section.
4. I work at a day-care and tried to apply the methods I learned in class.
5. (Rather) I asked myself if my understanding was adequate enough that I felt comfortable with the information (and) if I could explain it to someone else and have them understand it.
6. Quite a bit.
7. Very often. Because upon entering this course I really dreaded the thought of teaching math because my confidence was low. That has since changed.

8. All the time.
9. Every day.
10. Quite frequently. I don't feel that my understanding would be adequate.
11. Sometimes.
12. All the time, because I though I knew math, but from the results of my tests and quizzes I don't think I know it as well as I thought.
13. Not very much. If given the chance I believe that I could do a better than satisfactory job.
14. Pretty frequently because I thought of how I could use what we were learning and how to apply it to teaching and if it was really necessary.
15. All the time, but I know my understanding has greatly increased. I am more mathematically confident.
16. With almost every quiz or assignment.
17. I always ask and I'm always trying to be better.
18. A couple of times.
19. Often. It scares me to think that if I don't have a complete understanding for the subject how the heck am I supposed to teach it to kids.
20. Whenever I learned something new.

Number of hours you studied per week for this class
3, 3, 1-2, 4, 5-8, 3, 2-4, 6, 2, 10, 2, 5, 2-3, 2, 4-6, 1, 5, 2-3, 2

Number of hours you work per week
20, 20, 4, 15, 19, 0, 25, 20, 15, 8, 25, 10-15, 30+, 30, 30-35, 20-25, 32, 7--10, 20

Attended regularly?
All yes

Grade expected
C+, C, B, B or A-, B or B+, C, B+, B, D, B, C, C-, B, B-B+, B, B, B, or C, Passing, B- or C+, B

Highest grade received in a high school math class
B, A, A, A, A, C, A, A, D, A, B, B, A, A, B, A, A, B, B, A

Highest level math course in high school
Pre-calculus, algebra, calculus, trig, geometry, algebra II, 3 years, trig, algebra 1, inter. algebra, trig, trig, algebra II, calculus, geometry, geo-trig, algebra II and trig, algebra II and geo, algebra II, inter. algebra

APPENDIX B.7

RATIONAL NUMBER QUESTIONNAIRE
(RNQ)

MATH 211

SPRING 1990

QUESTIONNAIRE TO ASSESS UNDERSTANDING OF RATIONAL NUMBERS AND PROPORTIONAL REASONING CONCEPTS

YOUR NAME____

DIRECTIONS: You will find some of the problems on these pages easy, and others quite difficult. **Do the best that you can.** You **will not be graded** on this written test, but items like these will be used later in the term to evaluate the class' growth in understanding of these important concepts and skills.

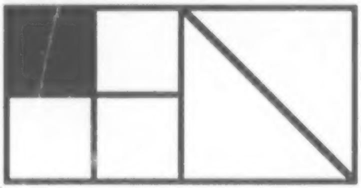
Please **do not use any other paper**. You should have sufficient room on these pages to do all the work you find necessary. [More space is provided on the original.] Other directions will be given as needed.

For the purpose of this test, calculators are not appropriate and should not be used.

1. What is 40% of 50? __________ 2. What is 150% of 6?______________

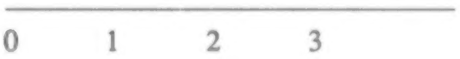
3. 10 is what percent of 30?________ 4. 15 is 75% of? ____________________

5. If these squares □□□□□□ represent $\frac{3}{2}$ of some whole amount, how many squares are in the whole? _________

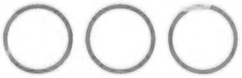


6. What fractions of this rectangle is shaded? ____________

7. Put a dot where $\frac{2}{3}$ is on this number line.

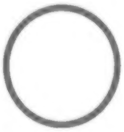


8. Show how these pizzas could be fairly shared by 8 people:
How much will each person get? ___________



9. Show how this bar of candy could be fairly shared by 4 people: How much will each person get? ___________

10. Shade $\frac{3}{8}$ of



11. In each of the following, circle the largest number:

a) $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$

b) $\frac{3}{4}$, $\frac{5}{16}$, $\frac{9}{10}$

c) 3.43 3.158 3.2

d) $\frac{1}{5}$ 0.3 $\frac{2}{12}$

e) 0.5 $\frac{13}{24}$ 0.46

f) $\frac{1}{9}$, $\frac{2}{3}$, $\frac{3}{11}$, $\frac{14}{28}$

12. Estimate the product of .47 x 2.1743 by circling the best estimate.

a) 1   b) 9   c) 100   d) 0.981921   e) 0.01

13. Estimate the sum: $\frac{12}{13} + \frac{7}{8}$ by circling the best estimate.

a) 1   b) 2   c) 19   d) 21   e) None of a-d

14. Choose the best answer: What happens to $\frac{3}{16}$ if the numerator is increased and the denominator is decreased?________

a) the value of the fraction increases
b) the value of the fraction decreases
c) the value of the fraction stays the same
d) there is not enough information to tell

15. Choose the best answer: What happens to $\frac{3}{16}$ if the numerator and denominator are both increased? ______________

a) the value of the fraction increases
b) the value of the fraction decreases
c) the value of the fraction stays the same
d) there is not enough information to tell

16. a) Find a decimal number, if possible, between 0.7 and 0.8 ___________

b) Find a fraction number, if possible, between 0.7 and 0.8 ___________

17. a) Find a fraction, if possible, between $\frac{1}{4}$ and $\frac{1}{5}$ ______________

b) Find a decimal, if possible, between $\frac{1}{4}$ and $\frac{1}{5}$ ______________

18. Place the decimal point in the answer:

a) 4.5 x 51.26 = 0 2 3 0 0 6 7 0 0 0

b) 30 ÷ .6 = 0 0 5 0 0 0 0 0

19. Fill in the missing numbers to make these true statements:

a) $\frac{4}{6} = \frac{6}{\quad}$ c) $\frac{15}{9} = \frac{\quad}{12}$

b) $\frac{3}{8} = \frac{\quad}{12}$ d) $\frac{8}{15} = \frac{\quad}{5}$

Directions For problems 20 - 25, you do not need to work the problem. Just show how you would find the answer using the given numbers.

Example: Jack drank $\frac{1}{2}$ glass of milk, then drank $\frac{1}{4}$ glass of milk.

How much did he drink in all? **Answer:** $\frac{1}{2} + \frac{1}{4}$

20. Each bead is $\frac{5}{8}$ of an inch long. If 40 beads are strung on a necklace, how long is the necklace?

21. A recipe calls for $2\frac{1}{2}$ cups of flour. If Karen has 9 cups of flour, how many recipes can she make?

22. A pound of cheese costs $2.46. How much does 0.78 pounds of the same kind of cheese cost?

23. Maria takes 3 minutes to read a page. Vaneta takes $\frac{3}{8}$ as long to read a page as Maria. How long does it take Vaneta to read a page?

24. Jim bought $4\frac{1}{2}$ pounds of coffee and Sally bought $2\frac{1}{2}$ pounds of coffee. Jim bought how many times as much as Sally?

25. Val ate $\frac{2}{3}$ pound of candy, and Ed ate $\frac{3}{8}$ pound of candy. Who ate more candy, and how much more?

26. In the following items, circle the case where the orange juice is **stronger**, that is, has more orange flavor, when the orange juice concentrate (ojc) and water are mixed **or** circle Same.

Example: A: 2 ojc, 1 water B. 1 ojc, 2 water C. Same

a) A. 2 ojc, 1 water B. 4 ojc, 3 water C. Same

b) A. 2 ojc, 1 water B. 3 ojc, 3 water C. Same

c) A. 1 ojc, 3 water B. 2 ojc, 5 water C. Same

d) A. 2 ojc, 3 water B. 3 ojc, 4 water C. Same

e) A. 1 ojc, 2 water B. 2 ojc, 4 water C. Same

f) A. 5 ojc, 2 water B. 7 ojc, 3 water C. Same

g) A. 5 ojc, 7 water B. 3 ojc, 5 water C. Same

27. The ratio of red balls to blue balls is 2 to 5. There are 70 balls. How many are red?

28. Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this University." Use S to represent the number of students and P to represent the number of professors.

29. Write a story problem for which the answer could be represented by $\frac{1}{2} \times \frac{3}{4}$

Directions: Complete these problems. If you need workspace, use the space provided

30. $14\frac{3}{4} - 5\frac{1}{2} =$ ______________

31. $\frac{5}{8} + \frac{9}{18} =$ ____________

32. $3\frac{1}{2} \times 6\frac{2}{3} =$ ______________

33. $88 + 2.2 =$ _____________________

34. $3 + \frac{4}{3} =$ ___________________

35. $\frac{2}{3} + \frac{5}{6} =$ _______________________

36. $\frac{7}{8} + 2 =$ ____________________

37. $12 + \frac{1}{6} =$ ____________________

38. $3.4 + \frac{3}{4} =$ __________________

39. $\frac{8}{16} \times \frac{5}{7} =$ ____________________

40. $\frac{5}{16} + \frac{7}{24} =$ ________________

41. $97 - 0.4 =$ _____________________

42. $3.2 \times 2.4 =$ _________________

43. $\frac{1}{2} + 0.5 =$ _____________________

44. $7.2 + 12 =$ __________________

45. Simplify: $\frac{\frac{4}{12}}{\frac{1}{3}} =$ ________________

46. Simplify: $\frac{4}{5/4}$ _____________

47. $1000 + 10.001 + 0.01 =$ ___________

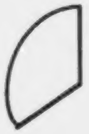
APPENDIX B.8

MATH 211
INTERVIEW
FORMS AND DATA

Interview items for the beginning of the course:

Part I: Rational numbers and proportional reasoning.

1. 12 is $\frac{3}{4}$ of what number?
2. What is $\frac{1}{2} \div 0.5$?
3. Approximately how many $\frac{4}{3}$'s are there in 3
4. Approximately how much is $5.8 \div 12$?
5. How would you do this problem: $16\frac{3}{4} - 10\frac{1}{2}$
6. Estimate: $152.34 + 49.718$
7. Estimate: $\frac{5}{6} + \frac{7}{8} + \frac{1}{15}$
8. Estimate: 40% of 92
9. Estimate: 12% of 204
10. Estimate: 0.52 x 789
11. Estimate: $\frac{2}{3}$ of 1.07
12. Tell me how you would do this problem mentally: 85 - 0.3
13. Tell me how you would do this problem mentally: 0.6 + 2.101 + 4.42
14. A candy factory packages 100 bars to a box. The factory has 34,721 bars on hand. How many boxes can be filled?
15. If xxx xxx xxx is $\frac{3}{4}$ of the whole, what is the whole?
16. Where does the decimal point go in 0.23 x 9.04 = 0 0 0 0 2 0 7 9 2 0 0 0 0?
17. This is $\frac{1}{2}$:



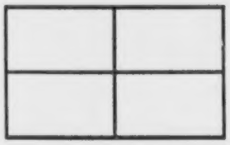
Draw the whole.

18. Order these from smallest to largest:

$\frac{7}{8}$, 0.31, $\frac{1}{3}$, 0.2, 0.75, $\frac{1}{4}$

19. Make a drawing to represent $\frac{1}{2} + \frac{3}{4}$

20. Shade in $\frac{2}{3}$ of this rectangle:



21. "If Nick drove fewer miles in more time than he did yesterday, his driving speed is:

(a) faster than yesterday (b) slower than yesterday

(c) the same as yesterday, or (d) there is not enough information to tell

Part II: Probability and Statistics

22. Discuss the relative likelihood of getting these three hands, in this order, during a game of cards:

(a) Three of hearts, four of hearts, five of hearts, six of hearts, seven of hearts.

(b) King of diamonds, three of spades, four of clubs, queen of spades, two of hearts.

(c) Ace of hearts, ace of spades, ace of diamonds, ace of clubs, jack of spades.

23. All families of six children in a city are surveyed. In 72 families the *exact order* of births of boys and girls was GBGBBG. What is your estimate of the number of families surveyed in which the *exact order* was BGBBBB ?

24. (a) What does it mean for a weather forecaster to predict a 70% chance of rain?

(b) What would you conclude if it does *not* rain?

(c) Suppose the weather prediction was for 70% rain, for 10 days in a row. Suppose there was no rain on three of these days. What would you say about the forecaster's predictions?

(d) What does a 50% prediction mean?

25. In three tosses of a fair coin, heads turned up twice and tails turned up once. What is the probability that heads will turn up on the fourth toss?

26. Suppose you knew that the pool of 100 persons contained 30% engineers and 70% lawyers. A certain person is drawn at random from this set of 100 people. The person is male, 45, conservative, ambitious, and has no interest in political issues. Which is more likely, that the person is a lawyer, or that the person is an engineer?

27. A particular woman is known to be bright, single, 31, outspoken, and concerned with issues of social justice. Which is more likely:
The woman is a bank teller, or,
The woman is a bank teller and belongs to a feminist organization.

28. Some employees in a company earn $5.00 an hour, and the rest of them earn $7.00 an hour. What is the average salary of the employees?

29. The real estate section of San Diego papers often report the median (middle) price of a home rather than the mean (average) price of a home. Why do you think they do this?

30. A student has an overall average of 50 (out of 100) on four exams. She has one more exam to take and wants to get an average of at least 70 on all five exams. Can she do it?

Math 211: Interview items for the end of the course:

Student name____________________

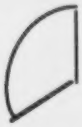
Part I: Rational numbers and proportional reasoning.

1. 12 is $\frac{3}{4}$ of what number?
2. What is $\frac{1}{2} \div 0.5$?
3. Approximately how many $\frac{4}{3}$'s are there in 3?
4. Approximately how much is $5.8 \div 12$?
5. How would you do this problem: $16\frac{3}{4} - 10\frac{1}{2}$
6. Estimate: 152.34 + 49.718
7. Estimate: $\frac{5}{6} + \frac{7}{8} + \frac{1}{15}$
8. Estimate: 40% of 92
9. Estimate: 12% of 204
10. Estimate: 0.52 x 789
11. Estimate: $\frac{2}{3}$ of 1.07
12. Tell me how you would do this problem mentally: 85 - 0.3
13. Tell me how you would do this problem mentally: 0.6 + 2.101 + 4.42
14. A candy factory packages 100 bars to a box. The factory has 34,721 bars on hand. How many boxes can be filled?
15. If the following is 3/4 of the whole, what is 2/3 of the whole?

```
If   xx        is 3/4 of the whole, what is 2/3 of the whole?
    x    xxx
    xx   x
```

16. A certain computer is broken and although it displays the correct digits, it does not show decimal points. It shows that 0.23 x 9.04 = 0 0 0 0 2 0 7 9 2 0 0 0 0. Where does the decimal point go?

17. This is $\frac{3}{4}$:



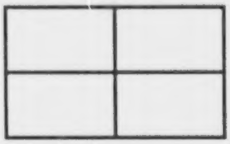
Draw the whole.

18. Order these from smallest to largest:

$\frac{7}{8}$, 0.31, $\frac{1}{3}$, 0.2, 0.75, $\frac{1}{4}$

19. Make a drawing to represent $\frac{1}{2} + \frac{3}{4}$

20. Shade in $\frac{2}{3}$ of this rectangle:



21. "If Nick drove fewer miles in more time than he did yesterday, his driving speed is:

(a) faster than yesterday (b) slower than yesterday

(c) the same as yesterday, or (d) there is not enough information to tell

Part II: Probability and Statistics

22. Discuss the relative likelihood of getting these three hands, in this order, during a game of cards:

(a) Three of hearts, four of hearts, five of hearts, six of hearts, seven of hearts.

(b) King of diamonds, three of spades, four of clubs, queen of spades, two of hearts.

(c) Ace of hearts, ace of spades, ace of diamonds, ace of clubs, jack of spades.

23. All families of six children in a city are surveyed. In 72 families the *exact order* of births of boys and girls was GBGBBG. What is your estimate of the number of families surveyed in which the *exact order* was BGBBBB ?

24. (a) What does it mean for a weather forecaster to predict a 70% chance of rain?

(b) What would you conclude if it does *not* rain?

(c) Suppose the weather prediction was for 70% rain, for 10 days in a row. Suppose there was no rain on three of these days. What would you say about the forecaster's predictions?

(d) What does a 50% prediction mean?

25. In three tosses of a fair coin, heads turned up twice and tails turned up once. What is the probability that heads will turn up on the fourth toss?

26. If $1.39 is the typical price for 8 different brands of a certain size bag of potato chips, what could the real prices be?

27. Beach balls in one store cost $5.00 each. In another store, the same balls cost $7.00 each. If you bought some balls in the first store and some in the second, what is the average price of the balls you bought?

28. The real estate section of San Diego papers often report the median (middle) price of a home rather than the mean (average) price of a home. Why do you think they do this?

29. A student has an overall average of 50 (out of 100) on four exams. She has one more exam to take and wants to get an average of at least 70 on all five exams. Can she do it?

Interview Data

1. 12 is $\frac{3}{4}$ of what number?

Pre	Post 4x4=16	9,then 16	$12 = \frac{3x}{4}$	$\frac{12}{x} = \frac{3}{4}$	12,1) $= \frac{3}{4}$	Inc.	Pre	Post
4 x 4 = 16	4						4	14
9, then 16	4	1	1				6	1
$12 = \frac{3x}{4}$	1		1	1			3	3
$\frac{12}{x} = \frac{3}{4}$							0	1
$\frac{12}{1} \times \frac{3}{4}$	3				1		4	1
Inc.	2		1			1	4	1

4 x 4 = 16 ---> Break 12 into 3 groups of 4, so 4 groups is 16, or 12 ÷ 3 = 4, 4 x 4 = 16.

9, then 16 ---> $\frac{12}{1} \times \frac{3}{4} = 9$, recognized answer too small, is said 16.

$12 = \frac{3x}{4}$ ---> Set up equation, 3x = 48, x = 16.

$\frac{12}{x} \times \frac{3}{4}$ ---> Set up proportion, 3x = 48, x = 16.

$\frac{12}{1} \times \frac{3}{4}$ ---> So answer is 9.

Inc. ---> Incorrect, or could not do.

2. What is $\frac{1}{2} \div 0.5$?

Pre	Post					
	Same as 1	$\frac{1}{2} \times 2$	$\frac{1}{2}$ of $\frac{1}{2}$	Inc.	Pre	Post
Same as 1	6		1	1	8	14
$\frac{1}{2} \times 2$	3	4			7	4
$\frac{1}{2}$ of $\frac{1}{2}$	4			1	5	1
Inc.	1				1	2

Same as 1---> Immediately recognized that $0.5 = \frac{1}{2}$, so quotient is 1.

1 x 2 ---> Worked out solution as $\frac{1}{2} \div \frac{1}{2} = \frac{1}{2} \times 2 = 1$ or $\frac{1}{2} \div \frac{5}{10} = \frac{1}{2} \times \frac{10}{5} = 1$.

$\frac{1}{2}$ of $\frac{1}{2}$ ---> $\frac{1}{2}$ of $\frac{1}{2}$ is $\frac{1}{4}$, or $\frac{1}{2}$ of 0.5 is .25

Inc. ---> other incorrect, e.g. 2.5

3. Approximately how many $\frac{4}{3}$s are in 3?

Number of Students, by Category of Response

			Post					
Pre	2	2+	2-	$2\frac{2}{3}$	3	Incorrect	Pre	Post
2	1	2	1	0	1	0	5	3
2+	2	4	1	0	0	0	7	8
2-	0	0	0	0	0	0	0	3
$2\frac{2}{3}$	0	0	0	0	0	1	1	2
3	0	1	0	1	0	0	2	3
Incorrect	0	1	1	1	2	1	6	2
							21	21

2: Approximately 2; not prompted as to more or less

2+: A little more than 2 (one in response to >/< prompt during posttest)

2-: A little less than 2 (two in response to >/< prompt during posttest)

$2\frac{2}{3}$: Exactly $2\frac{2}{3}$

3: $\frac{4}{3} \approx 1$, so there are approximately three $\frac{4}{3}$s in 3.

Methods used to reach 2 or 2+:
Count up: $1\frac{1}{3}$, $2\frac{2}{3}$, or $2 \times 1\frac{1}{3} = 2\frac{2}{3}$, so there are less than three $\frac{4}{3}$s in 3.
$3 \times 1 = 3$ and $\frac{4}{3} > 1$, so there are less than three $\frac{4}{3}$s in 3.

4. Approximately how much is 5.8 ÷ 12?

Post

Pre	6 ÷ 12	6 ÷ 12 late	2	4.8 ÷ 12	5.8 ÷ 12	Inc.	Pre	Post
6 ÷ 12	6	0	0	0	0	0	6	10
6 ÷ 12 late	0	1	0	0	0	0	1	1
2	0	0	1	2	0	0	3	3
4.8 ÷ 12	1	0	0	1	0	0	2	5
5.8 ÷ 12	3	0	2	1	1	0	7	1
Inc.	1	0	0	1	0	0	2	0

6 ÷ 12 ---> Immediately saw as about $\frac{1}{2}$ or 0.5

6 ÷ 12 late ---> Not immediate, but find answer is 6 ÷ 12 = $\frac{1}{2}$.

2 ---> 6 ÷ 12 is 2.

4.8 ÷ 12 ---> 4.8 ÷ 12 is 4, so between .4 and .5 or $\frac{5}{12}$.

1

5.8 ÷ 12 ---> 5.8 ÷ 12 is about 5, so .5, or 5.8 ÷ 12 is about .4

5. How would you do this problem: $16\frac{3}{4} - 10\frac{1}{2}$?

Number of Students, by Category of Response

Pre	Post $\frac{3}{4} - \frac{1}{2}$ 16-10	$16\frac{3}{4}$ ->17 comp	Lang reversed	Common denom	Improper fraction	$9\frac{5}{4}$ borrow	Answer only	Pre	Post
$\frac{3}{4} - \frac{1}{2}$; 16 - 10	0	0	1	0	0	0	0	1	12
$16\frac{3}{4}$ ->17, comp	0	0	0	0	0	0	0	0	1
Language reversed	0	0	0	0	0	0	0	0	3
Common denominator	4	0	1	2	0	0	1	8	4
Improper fraction	6	1	1	2	0	0	0	10	0
$9\frac{5}{4}$ (borrow)	2	0	0	0	0	0	0	2	0
Answer only	0	0	0	0	0	0	0	0	1
								21	21

$16\frac{3}{4}$ ->17, compensated in answer: User made error in compensating.

Language reversed (considered whole numbers, fractions separately): "16 from 10, get 6; subtract 3/4 from 1/2, get 1/4."

Common denominator: Changed $\frac{1}{2}$ to $\frac{2}{4}$ to subtract the fractions.

Improper fractions:

> "I feel more comfortable if they all have the same base." This student changed to improper fractions on the pretest, but could suggest subtracting whole numbers and fractions separately when asked to suggest another way to do the problem.
>
> On the Pretest one student who used this method was unsure if it is possible to subtract whole numbers and fractions separately, but said that would be a "sloppy way." The student would "work it out just to be sure." On the posttest this student subtracted the whole numbers and fractions separately, but reversed the language when explaining: "10 take away 16 is 6."

6. Estimate 152.34 + 49.718

Number of Students, by Category of Response

Post

Pre	152+50 202-202+	>200-201+	200	≈152+ ≈50 But error	Visualized Algorithm	Pre	Post
152 +50 202 - 202+	6	0	2	2	0	10	9
>200-201+	2	0	0	0	0	2	2
200	0	1	4	1	0	6	6
≈152+ ≈50 But error	0	0	0	0	0	0	4
Visualized Algorithm	1	1	0	1	0	3	0
						21	21

152 + 50: Answered "About 202," "ignore these (decimal parts)," or compensated--"202 point something."

Between 200 and 202: Added 152 + 49 then added a little for the decimals.

200: 150 + 50

Rounded to 152 and 50 but made an error: one student subtracted; two answered 252; one answered, "152 + 50 = 202, a little more, so 205 or 206."

Visualized the algorithm: "203.058."
"I'd have to write it down."

7. Estimate $\frac{5}{6} + \frac{7}{8} + \frac{1}{15}$

Post

Pre	<2	2	>2	CD	Pre	Post
<2	2	0	1	0	3	7
2	2	4	0	0	6	6
>2	0	1	1	0	2	8
CD	3	1	6	0	10	0

2 ---> 1 + 1 + 0. Can't tell is more or less than 2.

<2 ---> 1 + 1 + 0, $\frac{1}{15}$ not enough to make up the difference.

>2 ---> 1 + 1 + $\frac{1}{15}$, so >2.

CD ---> Can't do.

8. Estimate 40% of 92.

Number of Students, by Category of Response

Pre	Post 4 x 10%	$\frac{1}{2}$ or 50% > 40	$\frac{1}{2}$ or 50% <40	0.4x	4 x 90	Add	Pre	Post
4 x 10%	2(3*)	1	0	1	0(1)	0	4	2(3)
$\frac{1}{2}$ or 50% >40	0	4	1	1(2)	0	0	6	7
< 40	0	1	1	2	0	0(1)	4	2
0.4x	0	1	0	3	0	0	4	8(9)
4 x 90	0	0	0	0	2	0	2	2(3)
Add	0	0	0	1	0	0	1	0(1)
							21	21(25)

* Includes spontaneously given alternative, or response to "Any other way?"

4 x 10% = calculated 10%, then quadrupled

$\frac{1}{2}$ or 50%

$\geq$ 40 = $\frac{1}{2}$ or 50% x 90, 92, or 100, adjusted so estimate $\geq$ 40

< 40 = $\frac{1}{2}$ or 50% x 90, 92, or 100, adjusted so estimate < 40

0.4x = 0.4 or 0.40 x 90, 92, or 100

4 x 90 = 4 or 40 x 90, 92 or 100

Add. = Showed evidence of additive strategy

9. Estimate: 12% of 204

	Post						
Pre	12% x 200	2 x 10%	10% of 200	50-25-12	Inc/couldn't	Pre	Post
12% x 200	2	0	0	0	0	2	5
2 x 10%	0	1	1	0	0	2	2
10% of 200	2	1	8	0	1	12	12
50-25-12	0	0	2	0	0	2	1
Inc/couldn't	1	0	1	1	0	3	1

12% x 200 = 12% x 200, or (10% x 200) + (2% x 200)

2 x 10% = 2 x (10% of 100)

10% of 200 = 10% of 200 or 204

50-25-12 = Calculated 50% of 204, then halved that, then halved that.

Inc/couldn't = Incorrect answer/wouldn't do without exact calculation

10. Estimate 0.57 x 789.

Pre	Post							
	$\frac{1}{2}$ x 800	$\frac{1}{2}$ x (700+)	$\frac{1}{2}$ x 789	front-end	1 x 789	couldn't	Pre	Post
$\frac{1}{2}$ x 800	8	0	1	0	0	0	9	15
$\frac{1}{2}$ x (700+)	2	0	0	0	0	0	2	1
$\frac{1}{2}$ x 789	2	1	3	0	0	0	6	5(6*)
front-end	1	0	0	0	0	0	1	0
1 x 789	0	0	0(1*)	0	0	0	0(1*)	0
Couldn't	2	0	1	0	0	0	3	0

$\frac{1}{2}$ x 800 = $\frac{1}{2}$, 0.5, or 0.50 x 800

$\frac{1}{2}$ x (700+) = e.g., $\frac{1}{2}$ x 700 + $\frac{1}{2}$ x 40

$\frac{1}{2}$ x 789 = $\frac{1}{2}$ x 789 or 790

Front-end = calculated 5 x 7, added zeros

1 x 789 = 1 x 789

Couldn't = Couldn't do/couldn't without exact calculation

11. Estimate: $\frac{2}{3}$ x 1.07.

Pre	Post						
	$\frac{2}{3}$ ->dec	$\frac{2}{3}$ fract	$\frac{1}{3}$ ->dec	alg	Inc/d.k.	Pre	Post
$\frac{2}{3}$ ->dec	3	0	0	0	0	3	8
$\frac{2}{3}$ fract	0	2	0	0	0	2	8
$\frac{1}{3}$ ->dec	2	4	0	0	0	6	1
alg	3	1	0	0	0	4	0
Inc/d.k.	0	1	1	0	4	6	4

$\frac{2}{3}$ ->dec = $\frac{2}{3}$ expressed as decimal or $\frac{66}{100}$

$\frac{2}{3}$ fract = fraction $\frac{2}{3}$ as answer

$\frac{1}{3}$ ->dec = $\frac{1}{3}$ expressed as decimal, then doubled

alg = (1.07 ÷ 3) x 2, or (1.07 x 2 ÷ 3

Inc./d.k. = Incorrect/didn't know/other

12. Tell me how you would do this problem mentally: 85 - 0.3.

Number of Students, by Category of Response

	Post							
Pre	$\frac{10}{10} - \frac{3}{10}$ 1 - 0.3	Just subtract	0.3 + ? = 85	Algorithm 85 point 0	Need p/p	Ignore place value	Pre	Post
$\frac{10}{10} - \frac{3}{10}$ or 1 - 0.3	2	0	0	0	0	0	2	6
Just subtract	1	0	0	0	0	0	1	1
0.3 + ? = 85	1	1	0	0	0	0	2	1
Algorithm	2	0	1	10	0	0	13	12
Need p/p	0	0	0	2	0	0	2	0
Ignore place value	0	0	0	0	0	1	1	1
							21	21

Algorithm:
"85 point zero; line up the decimals."
"Put a zero there (85.0), and then you just carry on."
"First I borrowed...."
"I'd say eighty-five point zero, bring this over...."
"I'd just have to visualize 85 point 0...."
"I lined up the decimals in my head."
(Seven incorrect answers using algorithm on pretest; two incorrect on posttest.)

0.3 + ? = 85:
"Think of 84. You have to make up the extra one. Point 7. 84 point 7."

Need p/p: "I need paper and pencil so I can look back."

Ignore place value: "About 8.2."
"55. Line up the point three."

13. Tell me how you would do this problem mentally: 0.6 + 2.101 + 4.42.

Number of Students, by Category of Response

Pre	Post: Decimals first	Left to right	2 #s at a time	Alg rt to left	Rightmost digits,...	Need p/p	Unclear	Pre	Post
Decimals first	0	0	0	0	0	0	0	0	3
Left to right	1	0	0	2	0	0	3	6	0
Add 2 numbers at a time	2	0	3	0	0	0	0	5	5
Algorithm right to left	0	0	1	2	0	0	1	5	5
Add rightmost digits together	0	0	0	0	0	1	0	1	1
Need paper and pencil	0	0	0	0	0	2	0	2	4
Unclear	0	0	1	0	0	1	0	2	3
								21	21

Add decimal parts first: 0.6 + 0.4 = 1; 4 + 2 + 1 = 7; 7.121
0.101 + 0.42 = 0.521; add 0.6, 1.121; so 7.121

Add left to right: "I did it left to right. I had a hard time visualizing all the 'place settings.'"
"2 + 4 is 6, then add the decimals."

Add two numbers at a time: "0.6 + 4.42 = 5.02; plus 2.101; 7.121."
"7.121. Point 6 jumped over to 4.42, which gave 5.02; then I added 2.101."

Algorithm: "In the first 'lane,' I have 1, in the second, 2, so 1,2; in the third, 11, so 1 point 21, 7."
"Line up the decimals or do it by the number of digits from the decimal point."
"It's easier if you line them over each other mentally. I'll start from the right...."

Add rightmost digits together(ignoring place value): "2 + 1 + 6 = 9,..."

Need paper and pencil: "Line them up and put zeros. ...I can't do it. I can't carry in my head."

14. A candy factory packages 100 bars to a box. The factory has 34,721 bars on hand. How many boxes can be filled?

Pre	Post							
	Moved d.p.	P-value aware	Alg. ok mental	Alg. inc mental	Need to write	Inc.	Pre	Post
Moved d.p.	6	0	0	0	0	0	6	10
P-value aware	0	2	0	1	0	0	3	3
Alg. Ok. mental	2	1	0	1	0	0	4	1
Alg. inc. mental	2	0	1	0	0	1	4	4
Need to write	0	0	0	1	1	1	3	1
Inc.	0	0	0	1	0	0	1	2

Moved d.p.= moved decimal point 2 places (or "knocked off 2 Os")

P-value aware = attacked by place value: 700÷100, 4000÷140,
30000÷100.

Alg. Ok, mental= 100 |34721 mentally, correct answer

Alg. inc. mental = 100 |34721 mentally, incorrect answer

Need to write = needed to write calculation down

Inc. = other incorrect

15. (Pre) If xxx xxx xxx is $\frac{3}{4}$ of the whole, what is the whole?

All correct except 1 student, who thought 9.

xx
(Post) If x xxx is 3/4 of the whole, what is 2/3 of the whole?
xx x

(unit = 12 Xs part)

All correct thinking except one student, who did $\frac{2}{3}$ of $\frac{3}{4}$.

One student wrote a proportion.

($\frac{2}{3}$ of whole part)

All correct except 2 students.

16. Where does the decimal point go in 0.23 x 0.904 = 0000207920000?

Number of Students, by Category of Response

Pre	Post Estimate	Count dp Ok ff ?	Count dp from 2	Count dp Rule	DK/ Guess	Count dp from left	Pre	Post
Estimate	0	0	0	0	0	0	0	6
Count dp Ok after ?	2	1	2	0	0	0	5	3
Count dp from the 2	1	1	1	0	0	0	4	8
Count dp Rule	0	0	4	1	1	0	6	2
Guess	1	1	0	0	0	0	2	1
Count dp from left	2	0	0	1	0	1	4	1
							21	21

Correct

Estimate: (Two estimates were incorrect)
"About a quarter of 9."
"25% of 9 would be about 2."
"23% of 9 would be 2 point something."

Counted decimal places, but corrected and could justify with an estimate following prompt:
" 20792." [I: Does it seem right?] "No, according to my math sense, 20% of 10 is 2, so 2.0792."
"Traditionally I'm taught four places from the right, 20792. It's probably more like 20. No, two point something, like 1/4 of 9. I like rules better."

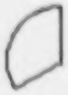
Counted the decimal places from the 2--recognized that the zeros were unnecessary:
"Four times three is 12, so I know (the answer) ended in two, and I counted over four places."
"(Started counting at the last 2) That's how I learned it, 2.0792."

Incorrect

Counted decimal places; because of the rule, unwilling to change answer when asked to estimate:
"20,792" [I: Is that ok?] "It sounds weird, but that's what I'd say."

Counted four places from the left or unsure which way to move decimal point:
"Four points to the right, 20792." [I: Reasonable?] "Oh, gosh. No, that's ok. Maybe I should carry over from the left. But I know to do it from the right." [I: Make sense?] "Less than half of nine. Like .2079 makes more sense."
"After the fourth zero. I was thinking you have to move it over. 2079.2. It shouldn't be that big. After the first 2. I'm not sure. I have to go four places somewhere. I did it from after the two. [I: Convinced?] "No, I need to write it down."

17. PRETEST: This is $\frac{1}{2}$:

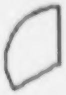


Draw the whole.

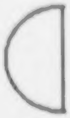
Number of Students, by Category of Response

Draw the same thing, the mirror image	13
Twice that	1
Drew correctly without explanation	5
Drew correctly but was unsure	1
Drew complete circle	1
Total	21

POSTEST: This is $\frac{3}{4}$:



or



Number of Students, by Category of Response

Divided given part in thirds; drew another third	16
Drew additional part without dividing given part in thirds	1
Drew correctly by comparing $\frac{3}{4}$ of circle	1
Trial and error until 4 equal parts, 3 in given	1
Incorrect	2
Total	21

18. Order these from smallest to largest:

$\frac{7}{8}$, 0.31, $\frac{1}{3}$, 0.2, 0.75, $\frac{1}{4}$

Number of Students, by Category of Response

	Post							
Pre	OK Fractions	OK Decimals	OK Both	OK No explan	Error	Frac/Dec separately	Pre	Post
OK, change to fractions	0	2	0	1	1	0	4	2
OK, change to decimals	0	4	0	3	0	0	7	10
Ok, change some to each	0	0	0	0	0	0	0	1
Ok, no explanation	1	0	0	0	0	0	1	4
Error/s	0	2	1	0	2	0	5	4
Frac/Dec separately	1	2	0	0	1	0	4	0
							21	21

OK, Fractions: Changed decimals to fractions where explained

OK, Decimals: Changed fractions to decimals, except $\frac{7}{8}$, which is "close to 1."

OK, Both: Changed 0.2 to a fraction; $\frac{1}{3}$ to a decimal.

No explanation given

Error/s: Common errors were $\frac{1}{4} > 0.31$; $\frac{1}{4} > \frac{1}{3}$; $0.31 > \frac{1}{3}$; $0.31 = \frac{1}{3}$.

Frac/Dec separately: Unable to order except fractions and decimals separately.

19. Make a drawing to represent $\frac{1}{2} + \frac{3}{4}$.

Number of Students, by Category of Response

				Post					
Pre	Pt/Wh A + A = Σ	Pt/Wh Σ only	A + A 5/4	Used #s vs drawng	A + A No sum	Draw ok Σ inc	Drew unit	Pre	Post
Part/whole A + A = Σ	2	3	0	0	0	0	0	5	8
Part/whole Σ only	2	1	0	0	0	1	1	5	6
Addend + addend 5/4 (vs 1 1/4)	0	1	1	0	0	0	0	2	1
Used numbers vs drawing to ans	2	0	0	1	0	0	0	3	1
Addend + addend Sum not given	1	0	0	0	0	0	1	2	1
Correct drawing Incorrect Σ	1	1	0	0	1	0	0	3	1
Drew unit first	0	0	0	0	0	0	1	1	3
								21	21

Part/whole model--Addend + Addend = Sum: Shaded circles or squares

Part/whole model-- Sum stated from drawn or shaded addends

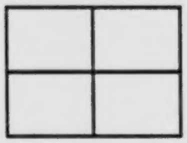
Part/whole model-- Sum of $\frac{5}{4}$ (vs $1\frac{1}{4}$) stated from drawn or shaded addends

Used numbers (vs drawing) to determine answer: Drawing used unequal wholes or did not clearly illustrate the problem, but the student answered $1\frac{1}{4}$.

Only addends drawn; sum not stated or drawn

Correct drawing using discrete or number line model, but sum given was incorrect

Drew unit first: "If this □ represents a quarter, ..."
"If this (circle) is a unit, ..."

20. Shade $\frac{2}{3}$ of this rectangle: 

Number of Students, by Category of Response

Pre	Post								
	Divide 3rds Shade 2	2 Of 3 2/3 of 4th	Estimate 2/3	12ths Shade 8	1 1/3 box = 1/3	Inc	> 1/2 < 3/4	Pre	Post
Divide into 3rds Shade 2	8	0	0	1	0	1	0	10	12
Shade 2 of 3 parts Shade 2/3 of 4th	0	0	0	0	0	0	0	0	1
Estimate 2/3 Ignoring divisions	1	0	1	0	0	0	0	2	1
Divide into 12ths Shade 8	1	0	0	2	1	0	0	4	3
1 1/3 box = 1/3 so 2 2/3 bx =2/3	0	0	0	0	0	0	0	0	1
Inorrect	1	0	0	0	0	1	0	2	3
Estimate: >1/2 and <3/4	1	1	0	0	0	1	0	3	0
								21	21

Divided into thirds (by drawing new lines or visualizing); shaded 2 parts:
"I didn't look at the divisions there. Mentally I broke it into thirds."
"[I could] not (shade 2/3) unless I changed the rectangle into thirds."
"Can I make my own lines? (Marks thirds, shades 2 parts)"

Shaded 2 entire parts and 2/3 of fourth part: "8/3 is 2 boxes and 2/3 of another."

Estimated 2/3, ignoring the divisions given: "I'd just totally ignore your lines... (Shades approximately 2/3). There's other ways--shade 2/3 of each box."

Divided into 12ths (divided each half into thirds); shaded 8 or shaded 2/3 of each half separately

Shaded 1 of 4 parts and shaded 1/3 of a second part --1/3 shaded; then shaded the remaining 2/3 of the second part and 1/3 of each of the remaining parts

Incorrect: Divided into 8ths and shaded 5 or 6
Divided into "thirds" unequally using diagonals for each half; shaded two of these parts

Estimated using given lines: "It's a little more than 1/2."

21. "If Nick drove fewer miles in more time than he did yesterday, his driving speed is:
(a) faster than yesterday (b) slower than yesterday
(c) the same as yesterday (d) there is not enough information to tell

Number of Students, by Category of Response

	Post						
Pre	b	a	c	d	Unsure/ Omitted	Pre	Post
b	9	0	0	2	0	11	16
a	1	0	0	1	0	2	1
c	1	0	0	0	0	1	0
d	3	1	0	1	0	5	4
Unsure or Omitted	2	0	0	0	0	2	0
						21	21

b: "He went fewer miles and he took longer."

a: "If he took more time, it could have been faster."

d: Additional information needed:
"You need to know how much he drove today."
"You need values."
"Did he go the same place?"

One student was unsure; the question was omitted in one interview.

22. Discuss the relative likelihood of getting these three hands, in this order, during a game of cards:

(A) Three of hearts, four of hearts, five of hearts, six of hearts, seven of hearts.
(B) King of diamonds, three of spades, four of clubs, queen of spades, two of hearts.
(C) Ace of hearts, ace of spades, ace of diamonds, ace of clubs, jack of spades.

Number of Students, by Category of Response

Post

Pre	Equally likely	B is easiest then C	then A	& A = C	B hardest	A hardest	Omitted	Pre	Post
Equally likely	1	0	0	1	1	0	1	4	11
B is easiest to get, then C, then A	3	1	1	1	0	0	0	6	3
B is easiest to get, then A, then C	1	1	0	1	0	0	0	3	2
B is easiest to get, A and C equal	5*	1	1	0	0	0	0	7	3
B is hardest to get	0	0	0	0	0	0	0	0	1
A is hardest to get (Calculated)	1	0	0	0	0	0	0	1	0
Omitted.	0	0	0	0	0	0	0	0	1
								21	21

*1--Theoretically equally likely, but would bet on B: "That's just theory; that's not real."

All hands equally likely: "Because each card is equally likely to come up."

B is easiest to get, then C, then A: "Hand B is more likely because it seems more random. Chances of getting A are very slim, much slimmer than the others."

B is easiest to get, then A, then C: "A and B are more likely than C; aces are hard to get. I think B is more likely than C; C has too many hearts."

B is easiest to get; A and C are equally likely (or relative likelihood of A and C not probed): "C is more likely. A would mean every other card was a heart. C-- it's hard to get four aces in a row unless you're really lucky. B is much more likely."

B is hardest to get

A is hardest to get (calculated)

Omitted: on one pre-interview

23. All families of six children in a city are surveyed. In 72 families the *exact order* of births of boys and girls was GBGBBG. What is your estimate of the number of families surveyed in which the *exact order* was BGBBBB?

Number of Students, by Category of Response

	Post								
Pre	≈ 72	< 72 ff prompt	> 72 ff prompt	Need more info	Don't know	5 - 6 families	Unclear	Pre	Post
≈ 72	0	0	0	0	0	1	0	1	6
< 72 following Prompt: < or >?	3	3	0	1	0	1	1	9	6
> 72 following Prompt: < or >?	0	1	0	0	0	0	0	1	0
Need more information	2	0	0	0	1	0	0	3	1
Don't know	1	2	0	0	2	1	1	7	3
5 or 6 families	0	0	0	0	0	0	0	0	3
Unclear	0	0	0	0	0	0	0	0	2
								21	21

≈ 72: "About the same; the chances of boys and girls are equally likely."
"There's an equal probability of boys and girls. My instinct would tell me < 72." [Interviewer: So you don't go by instinct any more?] "Not since this class."

< 72 following prompt (< or >?): "It would be a lot lower since there's only one girl."
"The odds are against so many boys in a row."

> 72 following prompt (< or >?):

Need more information: "You didn't give the number of families."
"Out of what?"

Don't know: "I know there's a 50% chance it's a boy or a girl."

5 or 6 families would have BGBBBB.

24a. What does it mean for a weather forecaster to predict a 70% chance of rain?
Number of Students, by Category of Response

	Post								
Pre	Based on data	30% not	Likely to rain	40% chance Should!	FC's opinion	Maybe raincoat	Omitted/ Unclear	Pre	Post
Based on data	1	1	0	0	0	0	0	2	6
30% chance won't rain	3	4	3	0	0	1	0	11	8
Good likelihood of rain	1	1	1	0	0	0	0	3	4
Should rain if even 40% chance	0	0	0	0	0	0	1	1	0
Forecaster's opinion	0	1	0	0	0	0	0	1	0
Probably rain; Maybe raincoat	1	0	0	0	0	0	1	2	1
Omitted/ Unclear	0	1	0	0	0	0	0	1	2
								21	21

Based on data--Under similar conditions in the past, it has rained 70% of the time:
"He studied the data."
"On past days with similar conditions, 70% of those days it rained."

There is a 30% chance it won't rain, 70% chance that it will.

There is a good likelihood, a great chance, that it will rain.

"It's going to rain. Usually when they say 40%, it's going to rain."

"In the forecaster's opinion, there is a 70% chance of rain."

[Interviewer: Would you take an umbrella/raincoat?]: "Maybe."
"It depends on what it looks like."

One unclear: "There are 70 chances of rain."
Omitted: Pre-one; Post-one

24b. What would you conclude if it does *not* rain?

Number of Students, by Category of Response

		Post					
		Got the 30%	Not all clds --> rain	WM Wrong	Unclear/ Omitted	Pre	Post
Pre	Got the 30%	8	0	0	0	8	14
	Not all clouds produce rain	2	0	0	0	2	0
	Forecaster wrong	4	0	5	0	9	6
	Unclear/ Omitted	0	0	1	1	2	1
						21	21

Got the 30%: "Well, he wasn't wrong, but the 30% came up."
The forecast was OK. "There was still a 30% chance it wouldn't rain."

"It was probably based on cloud information, and **not all clouds produce rain**."

Forecaster wrong: "The prediction was wrong."
"He's inaccurate."

24c. Suppose the weather prediction was for 70% rain, for 10 days in a row. Suppose there was no rain on three of these days. What would you say about the forecaster's predictions?

Number of Students, by Category of Response

Pre	Post About right	Very accurate	Right 70%	Forecasts uncertain	FC wrong	Better than (b)	Omitted/ DK	Pre	Post
About right	4	2	1	0	0	0	1	8	8
Very accurate	0	3	0	0	0	0	1	4	7
Right 70% of the time	1	0	1	0	0	0	0	2	3
Forecasts are uncertain	0	0	0	0	0	0	0	0	1
Forecaster wrong	1	2	0	1	0	0	0	4	0
Better than prdiction in (b)	0	0	1	0	0	0	0	1	0
Omitted/Unclear Don't know	2	0	0	0	0	0	0	2	2
								21	21

About right: "Not absolutely right, but pretty close."
"Good prediction."
"He's a pretty good forecaster."

Very accurate: "He's 70% right; no, he's 100% right."
"He was right on target."

Right 70% of the time: "He's correct more than half the time."
"Correct 7 days; wrong 3."

Forecasts uncertain: "Well, it's just a forecast, so you won't know for sure."

Weatherman was wrong: "He overestimated; 70% is so large, it should have rained every day."
"It would be very inaccurate."
"Need a new forecaster."

Prediction was better than the prediction in the previous part, part (b) of the question.

Omitted: Pre-1; Post-1
Unclear: Post-1
Don't know: Pre-one

24d. What does a 50% prediction mean?

Number of Students, by Category of Response

Pre	Post: 50-50 Data basd	same conds rain 50%	Rain 5 of 10 days	50-50 No explan	WM not know	Omitted	Pre	Post
50-50, Data based	4	0	0	0	0	0	4	12
Same condits rains 50%	0	0	0	0	0	0	0	2
Rain 5 of 10 days	1	0	0	1	0	1	3	1
50-50, No explanation	4	2	1	1	1	0	9	4
Weatherman doesn't know	3	0	0	1	0	0	4	1
Omitted	0	0	0	1	0	0	1	1
							21	21

50-50, Report based on data (following interviewer's question):
"50-50 chance. [Does that mean the forecaster doesn't know?] "No, there has to be some data there for him to predict rain at all."
"He definitely has to base it on information. It's better than no forecast at all."

Same conditions produced rain 50% of the time in the past

Will rain 5 of 10 days--would take an umbrella or raincoat (in response to interviewer's question)

50-50 chance of rain (Unsure or not asked if prediction based on data): "It could go either way. If a weatherman says that ten days in a row, he blows it."

Weatherman doesn't know: "I wouldn't go by his forecast."

25. In three tosses of a fair coin, heads turned up twice and tails turned up once. What is the probability that heads will turn up on the fourth toss?

		Post					
		$\frac{1}{2}$	$>\frac{1}{2}$	$<\frac{1}{2}$	d.k./n.s.	Pre	Post
Pre	$\frac{1}{2}$	11	0	0	0	11	4
	$>\frac{1}{2}$	2	1	2	0	5	1
	$<\frac{1}{2}$	0	0	0	0	0	3
	dk./n.s. 1	0	1	1*	3	1	

* This student missed most of the classes on probability.

$\frac{1}{2}$ = $\frac{1}{2}$ or 50% or 50-50

$>\frac{1}{2}$ = 75%, $\frac{3}{4}$, 2 to 1, or "H more likely"

$<\frac{1}{2}$ = $\frac{1}{16}$ or "less than half"

26. (Pre only)
Suppose you knew that the pool of 100 persons contained 30% engineers and 70% lawyers. A certain person is drawn at random from this set of 100 people. The person is male, 45, conservative, ambitious, and has no interest in political issues. Which is more likely, that the person is a lawyer, or that the person is an engineer?

Number of Students, by Category of Response

Numbers unimportant. Engineer since pi, cons.	13
70% lawyers, so more likely a lawyer	3
Unsure when consider #s and other info	4
Don't know	1
Total	21

Numbers less important than other information:
"Engineer; I don't see a lawyer as conservative."
"He's an engineer; he's not interested in political issues." [Interviewer: Are the numbers irrelevant?] "Since I know there are both, I know he could be an engineer."
"Lawyers aren't as conservative, more interested in political issues." [Interviewer: Do the numbers matter?] "No, it's a random sample."

Since there are 70% lawyers, the person is more likely a lawyer:
"Engineer; a lawyer has to be interested in political issues... The chances are if 70% are lawyers, you'd be more likely to get a lawyer." [Interviewer: Which do you trust more (numbers or other information)?] "Percentage."
"There's more chance of picking a lawyer."

Unsure when consider both numbers and other information:
"It's a double-edged question."
"He's probably an engineer (because of the political issues).... Mathematically I'd say he's a lawyer."

27. (Pre only)
A particular woman is known to be bright, single, 31, outspoken, and concerned with issues of social justice. Which is more likely:

The woman is a bank teller, or
The woman is a bank teller and belongs to a feminist organization?

Number of Students, by Category of Response

Bank Teller	0
Bank teller and member of feminist organization	17
50-50	1
Omitted	3
Total	21

Bank teller and member of feminist organization:
"She could be just a bank teller." [Interviewer: Which is more likely?] "That she is with a feminist group."
"Is she attractive? I shouldn't have said that. Bank teller and member of a feminist organization... outspoken, single, 31--the whole thing says it all."

50-50:
"There's a 50-50 chance. I lean toward being in a feminist group. I still say 50-50."

26. (Post only)
If $1.39 is the average price for 8 different brands of a certain size bag of potato chips, what could the real prices be?

Number of Students, by Category of Response

Some higher, some lower; possible 7 cost < $1.39	8
≥$20 possible	6
Impossible to have 7 priced > $1.39	2
Need same number > $1.39 as < $1.39	1
$1.39 each	1
Highest must be < $3	2
Lacks concept of average	1
Total	21

Some higher, some lower; possible 7 cost more or less than $1.39 (following interviewer question); if asked, answered that $20 would be too much.:
"(There could be seven priced at more than $1.39) if there was one that was really low."

≥$20 possible: "It could be anywhere above or just a little bit below."
[Interviewer: Could one be $20?] "Yes, if another was like 50¢. It can't be infinite; there is a limit at the other end, and you can't go below zero."

Need same number > $1.39 as < $1.39: "You could have one very high and one very low. If some are very high, the same number would be very low." [I: Could there be 5 high and 3 low?] "No."

$1.39 each: "(The prices would all be) the same. [I:Are there other possibilities?] "It might depend on the size." [I: If they're all the same size?] "I would say, 'the same.'"

28. (Pre) Some employees in a company earn \$5.00 an hour, and the rest of them earn \$7.00 an hour. What is the average salary of the employees?

only \$6: 1
# employees matters: 13
no, or not sure whether # employees matters: 4
incorrect or unclear: 3

27. (Post) Beach balls in one store cost \$5.00 each. In another store, the same balls cost \$7.00 each. If you bought some balls in the first store and some in the second, what is the average price of the balls you bought?

Only \$6: 0
# balls matters: 14
no, or not sure whether # balls matters: 6
incorrect: 1

Aware \$ 5 ≤ average ≤ average # 7? (not asked of all)

Yes: 8 out of 9
No: 1 out of 9

29 (28 post) The real estate section of San Diego papers often report the median (middle) price of a home rather than the mean (average) price of a home. Why do you think they do this?

CB. pre	"Might look more attractive." [If relatives were moving to town, which would you tell them?] "If wanted them to come, would give median. The average is really high. The median is more attractive."
CB. post	"Because the median price would be an actual price, and the average takes outliers into account." [Relative?] "Median"
Jc. pre	"... is it like, to get you interested?" [Relative?] I'd probably tell them the average.
Jc. post	"Because the average would be high. Scare customers."
Mc. pre	"Because the middle price is lower than the average." [Relative?] "I would give the average. It's more what people are paying."
Mc. post	"I guess the median price is lower than the mean."
PE. pre	"The median is a little less, probably. "[Relative?] "I'd tell a relative the average."
PE. post	"The median could be higher than the average. People would think houses are cheaper." [Relative?] "I'd tell the mean."
DF.pre	"Makes the prices look lower. I think the middle would be smaller." [Relative?] "...middle price."
DF. post	"Probably because the median is lower than the mean." [Relative?] "I'd probably give the mean."
HG. post	"...It would be deceptive, I think. The middle is irrespective of the average. The convenient way may not tell the whole truth."
AG. post	"It gives you an idea of the exact market, an exact price. If you said mean, there'd be many homes around the same price." [Relative?] "I'd give them the mean. The middle will gave you the whole market. The mean will give part." ...[If I said the median was $208,000?] That would mean the prices go from $0 to $416,000.
RH. pre	"Because some houses are very expensive. So the median is an average middle-class price." [Relative?] "I'd say middle to a relative. It could be more in their price rate."
RH. post	"Because some houses in La Jolla would be outrageous and that could set off the average." [Relative?] "I would tell them the mean. [So you'd want all those La Jolla...?] "No, I wouldn't want these houses considered..." I'd tell them the median.
LL. post	"Because it's not affected by extremes. They don't want you to see how expensive it is. [Relative?] "I'd give the mean. The middle number is not affected by extremes. It's more honest."

SM, pre	"Well, the average is probably a lot higher than the median. "I think they do it to lure the customers in." [Relative?] "Give her the mean, cause it's quite expensive around here, so I'd want her to know."
KM, post	"The mean is influenced by outliers, but not the median. The median looks better. More people look like they can afford it." [Relative?] "The median. No, the mean it's more realistic."
LT, pre	"It's probably better estimate... The average gives a broad range. The median is probably a better estimate, and gives the public a better estimate."
LT, post	"The average takes into account expensive homes and cheap homes. The median targets an area where more people would be able to (afford)." [Relative?] "I'd give the median."
CW, post	"... The median is less swayed by outliers."

30. (29 post). A student has an overall average of 50 (out of 100) on four exams. She has one more exam to take and wants to get an average of at least 70 on all five exams. Can she do it?

Post

Pre	350 tot.	100 on last	$\frac{50+90}{2}$	yes but	?	Pre	Post
350 tot.	1	2	1	0	0	4	5
100 on last	0	3	0	0	0	3	8
$\frac{50+90}{2}$	0	1	0	2	0	3	2
yes but	1	1	0	1	0	3	3
?	3	1	1	0	0	5	0

350 total = reasoning based on needed 350 total

100 on test = reasoning based on 100 on last test

$\frac{50+90}{2}$ = reasoning based on $\frac{50+90}{2} = 70$

yes but = thought yes/no but couldn't prove

? = couldn't decide

APPENDIX C.1

Demographics and Attitude Presurveys

NS412C

Please note that this is for research purposes only, and individual responses will be kept confidential.

DEMOGRAPHICS SURVEY

1. Name ______________________________
2. Social Security number ____________________
3. Age _____
4. _____ Female; ____ Male (please check one)
5. Ethnic background ____________________
6. Year in college (1st semester, Junior, etc.) ______________________
7. Overall G.P.A. _________
8. S.A.T. scores (approximate if necessary) _______ Math; ______ Verbal
9. Semester hours of college biology ______________
10. What college(s) have you attended other than SDSU? _________________

____

11. Do you have employment outside of school? ___ yes; ___ no (please check one)
12. If you answered "yes" to question #11, please indicate where and how many hours/week work.

Place of employment ______________________________________

Hours/week on the average __________

13. Do you have work experience related to science? ___ yes; ___ no
14. If you answered "yes" top question #13, please indicate what type of work and approximately how long you were employed.

Type of work ____

____

____

Approximate length of employment _______________________

15. Do you have work experience related to teaching? ___ yes; ___ no
16. If you answered "yes" top question #15, please indicate what type of work and approximately how long you were employed.

Type of work ____

____

____

Approximate length of employment _______________________

17. If you are willing, please briefly describe your home living arrangements (children, parents, roommates, status as sole supporter, etc.)

____

____

____

____

SCIENCE TEACHING AND LEARNING SURVEY (STLS)

1) Use a #2 pencil to mark your answers; mark only on the computer answer sheet.
2) Write "STLS" in the blank marked "Test" on your computer answer sheet.
3) Write your social security number on the computer answer sheet and fill in the corresponding circles.
4) THIS SURVEY IS FOR RESEARCH PURPOSES ONLY! Please answer every question as honestly as you can. Your responses are confidential and no one will know your individual responses.

****

This scale consists of a series of statements. Remember there are no "right" or "wrong" answers to these statements. They have been designed to allow you to indicate how much you agree or disagree with the ideas expressed.

As you read each statement, write down your first reaction to it. If you strongly agree, blacken circle A on your computer answer sheet. If you agree, but with reservations, that is, you do not agree completely, blacken circle B. If you disagree with the idea, indicate the extent to which you disagree by blackening circle D for disagree or circle E for strongly disagree. But if you are truly undecided, that is, nether agree or disagree, blacken circle C.

A) STRONGLY AGREE B) AGREE C) UNDECIDED D) DISAGREE
E) STRONGLY DISAGREE

****

1. I think I will feel uncomfortable teaching science.
2. The acquisition of scientific processes (classification, simple experimenting, metric measurement, observations, etc.) is important in the elementary/middle level classroom.
3. I fear that I will not be able to teach science adequately.
4. Teaching science takes too much time.
5. I think I will enjoy the laboratory period in the science courses that I will teach.
6. I have difficulty understanding science.

7. I feel confident about teaching the science content in grades 5-6.

8. I feel confident about teaching the science content in grades 7-8.

9. I feel confident about teaching the science content in grades 8-9.

A) STRONGLY AGREE B) AGREE C) UNDECIDED D) DISAGREE

E) STRONGLY DISAGREE

10. I would be interested in working with a middle school science program involving experiments.

11. I dread teaching science.

12. I am not afraid of demonstrating scientific phenomena before my students.

13. I am not looking forward to teaching science.

14. Science should be incorporated within several of the other subjects.

15. I would enjoy helping students construct science equipment.

16. As I teacher I will be willing to spend time setting up equipment and gathering materials for a lab.

17. I am afraid that my students will ask me questions that I cannot answer.

18. I look forward to exploring questions about science with my students.

19. Science is very interesting to me.

20. Science makes me feel secure, and at the same time it is stimulating.

21. Science makes me feel uncomfortable and insecure.

22. In general, I have a good feeling toward science.

23. I approach science with a feeling of hesitation.

24. It makes me nervous to even think about doing a science experiment.

PERCEPTIONS OF SCIENCE AND SCIENTISTS (PSS)

1) Use a #2 pencil to mark your answers; mark only on the computer answer sheet.
2) Write "PSS" in the blank marked "Test" on your computer answer sheet.
3) Write your name on the computer answer sheet and fill in the corresponding circles.
4) THIS SURVEY IS FOR RESEARCH PURPOSES ONLY! Please answer every question as honestly as you can... your responses are confidential and no one will know your individual responses.
5) When you have finished, please return both this form and your computer answer sheet to your teacher. YOUR TEACHER WILL NOT BE EXAMINING YOUR RESPONSES!

****

This survey is designed to learn how you view science and scientists. There are NO right or wrong answers. Use the following scale to indicate how much you agree or disagree with these statements about science and scientists:

A) STRONGLY AGREE B) AGREE C) UNDECIDED
D) DISAGREE E) STRONGLY DISAGREE

****

1. Scientific research is often done outdoors.

2. Working in science is very exciting.

3. Science is very difficult to understand.

4. Working in science is very frustrating.

5. Being a scientist is very personally rewarding.

6. Scientific work is very tedious.

7. By learning about science, I can see how things in nature all "fit together."

8. Scientific research can harm as well as help people.

9. Scientific work is very important to all people.

A) STRONGLY AGREE B) AGREE C) UNDECIDED
D) DISAGREE E) STRONGLY DISAGREE

****

10. Science is mostly unrelated facts which you have to memorize.

11. Scientific research is usually done in laboratories.

12. Most science work is done using mice, rats, or chemicals.

13. Learning about science is easy.

14. Science is often boring.

15. Science is fun to think about.

16. Science is very interesting.

17. Scientists often work as a team to solve problems.

18. Scientists usually wear white laboratory coats.

19. Most scientists have time to enjoy their families and participate in family activities.

20. People who work in science careers don't have the opportunity to travel much in their work; they spent most of their time at their workplace.

21. Most scientists are geniuses.

22. Many scientists have few interests outside their work.

23. Scientists often are "work-a-holics."

24. It takes many years of college to be able to get a science-related job.

25. Most people who do science like to work alone.

26. Many science careers require only 2-4 years of schooling after high school.

A) STRONGLY AGREE B) AGREE C) UNDECIDED
D) DISAGREE E) STRONGLY DISAGREE

****

27. Most scientists spend so much time on their work, they have little time left to spend with their families.

28. Like artists and musicians, scientists are very creative in their work.

29. Scientist usually talk about their work only to other scientist.

30. Only brilliant people work in science careers.

31. Most scientists spend too much time at work.

32. Scientists work very hard.

33. Scientists often develop hobbies in music and art.

34. You have to be a "straight-A" students to make it in a science career.

35. A scientist must be able to talk with many different people.

36. Those who work in science careers frequently use their observation skills.

37. In order to work in science, one needs good writing skills.

38. Scientists are too busy analyzing things in nature to see the beauty in them.

39. Many times, scientists keep their work secret.

SCIENCES EXPERIENCES SURVEY (SES)

1) Use a #2 pencil to mark your answers; mark only on the computer answer sheet.
2) Write "SES" in the blank marked "Test" on your computer answer sheet.
3) Write your name on the computer answer sheet and fill in the corresponding circles.
4) THIS SURVEY IS FOR RESEARCH PURPOSES ONLY! Please answer every question as honestly as you can... your responses are confidential and no one will know your individual responses.
5) When you have finished, please return both this form and your computer answer sheet to your teacher. YOUR TEACHER WILL NOT BE EXAMINING YOUR RESPONSES!

****

This survey is designed to learn how often you have participated in various science-related activities WHEN NOT REQUIRED FOR A CLASS. There are NO right or wrong answers. Use the following scale to mark your answers (Remember to count only those activities which were not required as a class assignment):

A) FREQUENTLY B) FAIRLY OFTEN C) SELDOM D) NEVER

****

HOW OFTEN HAVE YOU....

1. read science articles in magazines?

2. read science articles in newspapers?

3. watched science programs on television (for example, NOVA, The Body Human, etc.)?

4. read books about science or scientists?

5. talked about science topics with your friends

6. worked on science projects (for example, for a science fair, 4-H fair, etc)?

7. worked with science-related hobbies (for example, collected leaves or rocks, used a chemistry set)?

A) FREQUENTLY B) FAIRLY OFTEN C) SELDOM B) NEVER
(when not required for class)

8. listened to science or medical reports on news programs?

9. visited a planetarium or aquarium?

10. visited a weather station?

11. taken a tour of a sewage treatment plant?

12. visited a natural history or science/industry museum or a nature center?

13. taken a tour of a greenhouse or arboretum?

14. visited a mine or rock quarry?

15. taken a tour of an agricultural or other scientific research center?

16. explored the shore of an ocean?

17. walked in the desert?

18. viewed an archeological site?

19. taken care of farm animals?

20. walked through a cave?

21. visited the mountains?

22. visited a national park, nature preserve, or wildlife refuge?

23. taken a tour or worked in a veterinary clinic?

24. taken a tour through a pharmaceutical company?

25. taken a tour or worked in a hospital, medical, or dental facility?

26 examined a bird's nest closely?

A) FREQUENTLY B) FAIRLY OFTEN C) SELDOM B) NEVER
(when __not__ required for class)

27. taken something apart to see how it works?

28. performed a chemical experiment or used a chemistry set?

29. looked through a telescope at the night sky?

30. planted something and watched it grow?

31. found a fossil?

32. touched a snake or lizard?

33. fixed something mechanical?

34. fixed something electrical?

35. played with objects found in nature (for example, made a whistle from blades of grass of leaf rubbings with crayon and paper?

36. watched the birth of an animal?

37. viewed a solar or lunar eclipse?

38. worked on a car (for example, changing the oil, adding water to the radiator)?

39. cared for an unhealthy animal?

40. cared for an unhealthy plant?

SCIENCE ATTITUDES QUESTIONNAIRE (SAQ)

READ ALL DIRECTIONS BEFORE YOU START WRITING!

1) Use a #2 pencil to mark your answers; mark only on the computer answer sheet.
2) Write your name on the computer answer sheet and fill in the corresponding circles.
3) Write "SAQ" in the blank marked "Test" on your computer answer sheet.
4) THIS SURVEY IS FOR RESEARCH PURPOSES ONLY! Please answer every question as honestly as you can... your responses are confidential and no one will know your individual responses.
5) When you have finished, please return both this form and your computer answer sheet to your teacher. YOUR TEACHER WILL NOT BE EXAMINING YOUR RESPONSES!

****

This scale consists of a series of statements. There are no "right" or "wrong" answers to these statements. They have been designed to allow you to indicate how much you agree or disagree with the ideas expressed.

As you read each statement, write down your first reaction to it. If you strongly agree, blacken circle A on your computer answer sheet. If you agree, but with reservations, that is, you do not agree completely, blacken circle B. If you disagree with the idea, indicate the extent to which you disagree by blackening circle D for disagree or circle E for strongly disagree. But if you are truly undecided, that is, nether agree or disagree, blacken circle C.

Do not spend much time with any statements, but be sure to answer every statement. Remember, there are no "right" or "wrong" answers. The only correct answers are those that are true for you. Whenever possible, let your past experiences help you make a choice. Finally, remember that science includes not only biology, but also chemistry, physics, computer science, geology, etc..

A) STRONGLY AGREE B) AGREE C) UNDECIDED
D) DISAGREE E) STRONGLY DISAGREE

****

1. One of my highest priorities is to be an outstanding student in science.

2. Girls often have to work harder than boys do to earn good grades in science.

A) STRONGLY AGREE B) AGREE C) UNDECIDED
D) DISAGREE E) STRONGLY DISAGREE

****

3. Science is one of the most worthwhile and necessary subjects to take.

4. Being regarded as smart in science would be a great thing.

5. If I got the highest grade in science, I'd prefer no one knew.

6. I'm not good in science.

7. People would think I was a bookworm if I got As in science.

8. I'm not the type to do well in science.

9. It is more acceptable socially for a girl to ask a boy for help in science lab than for a boy to ask a girl for help in science lab.

10. Equal numbers of men and women have the potential to become great scientists.

11. When a question is left unanswered in science class, I continue to think about it afterward.

12. Women's science aptitude is as great as men's science aptitude.

13. Figuring out science problems does not appeal to me.

14. I have a lot of self-confidence when it comes to science.

15. Science is of little relevance to my life.

16. I do as little work in science as possible.

17. I like to be challenged by science problems I can't understand immediately.

18. For some reason, even though I study, science seems unusually hard for me.

19. I study science because I know how useful it is.

A) STRONGLY AGREE B) AGREE C) UNDECIDED
D) DISAGREE E) STRONGLY DISAGREE

****

20. I would rather have someone give me the solution to a difficult science problem than to have to work it out myself.

21. I don't like people to think I'm smart in science.

22. It is very important for me to get top grades in science.

23. I see science as a subject I will rarely use in my daily life as an adult.

24. I would expect a successful woman science student to be somewhat unfeminine.

25. I expect to have little use for science when I get out of school.

26. I am sure that I can learn science.

27. It is more important for boys to study science than for girls to study science.

28. I will use science in many ways as an adult.

29. Science is enjoyable and stimulating to me.

30. I am confident that I can get good grades in science.

PLEASE CHECK TO MAKE SURE YOU FOLLOWED ALL INSTRUCTIONS!

APPENDIX C.2

Content and Process Skills Pretests

NS412C

Natural Science 412C Pretest

Please mark all correct answers on your test sheet with a soft lead pencil, and do not mark on the test. Note that each question may have more than one correct answer.

This pretest will be scored but will not be counted toward your grade. There are 25 questions, 4 points each.

1. Yeast is a single-celled fungus that is used for making bread, beer, and wine. What molecule, produced by the yeast, is responsible for making bread rise and for creating bubbles in beer and wine?
   a) carbon dioxide b) lactic acid c) oxygen d) alcohol

2 Oral temperature typically drops with vigorous exercise. The specific mechanism of this temperature drop involves:

- a) increased respiration
- b) increased evaporation
- c) drop in core body temperature
- d) shunting of blood to the intestines

3. Limewater reacts with carbon dioxide to produce calcium carbonate, a white precipitate. When a green plant and a beaker of limewater are sealed in a baggy, a white crust typically forms on the surface of the limewater at night but not during the day because:

- a) respiration occurs at night but not during the day
- b) photosynthesis occurs at night but not during the day
- c) more CO_2 accumulates at night
- d) more CO_2 accumulates during the day

4. Mammals maintain a constant body temperature by . .

- a. altering the flow of blood to the body surface
- b. burning fuel (sugar) to produce heat energy.
- c. insulating their bodies from the environment via fat, fur, etc.
- d. shivering to raise body temperature through kinetic energy.

5. Which of the following cycle repeatedly through living and non-living phases on earth?
   a. carbon b. phosphorus c. nitrogen d. energy f. water

6. Detrivores (decomposers) include which of the following classes of organisms?
a. vultures b. bacteria c. fungi d. caterpillars

7. The process of dividing up a particular food supply so that species with similar requirements use the same resources in different areas, at different times, or in different ways, is called
a. interference competition
b. resource partitioning
c. character displacement
d. mimicry

8. The primary function served by the anther of a flower is
a. fertilization b. pollen production
c. germination d. seed formation

9. Which of the processes below occurs in meiosis (cell division to produce gametes) but NOT in mitosis (somatic cell division)?
a. two rounds of DNA replication
b. two cell divisions
c. exchange of parts between similar (homologous) chromosomes
d. pairing of similar chromosomes
e. formation of spindle fibers

10. Starch and cellulose are both built of glucose subunits but they differ in that
a. starch is easily broken down but cellulose is not
b. cellulose is easily broken down but starch is not
c. starch occurs in plants but cellulose occurs in animals
d. cellulose occurs in plants but starch occurs in animals
e. starch provides energy while cellulose provides strength

11. Chlorophyll acts in photosynthesis by
a. regulating chemiosmosis in the chloroplast
b. absorbing visible light rays in all but the green spectrum
c. capturing light energy
d. synthesizing glucose

12. Which processes are important in transpiration (water flow) in plants?
a. evaporation b. cohesion c. adhesion
d. capillary action e. root pressure

13. Which particles are found in the nucleus of an atom?
a. protons b. electrons
c. photons d. neutrons

14. How much larger is a cell nucleus compared to a nucleus of an atom? The cell nucleus is
a. 10X larger than an atomic nucleus
b. 100X larger than an atomic nucleus
c 1000X larger than an atomic nucleus
d. more than 10,000X larger than an atomic nucleus

15. What advantages does fat have compared to carbohydrates for energy storage? Fat . . .
a. provides more efficient energy storage
b. contains more energy per pound
c. is more soluble in water
d. is more easily converted into sugar

16. Which organs produce digestive enzymes?
a. pancreas b. thymus c. liver d. thyroid e. spleen

17. Villi and microvilli are found in the
a. kidney b. liver c. small intestine
d. spleen e. nasal passage

18. The kidney is responsible for
a. filtration of plasma
b. production of FSH (follicle stimulating hormone)
c. reabsorption of water and dissolved substances
d. helping to regulate the ion balance in the blood

19. The number of known endocrine glands in the human body is about
a. 10-20 b. 20-100 c. 100-200 d. 1000-2000

20. The movement of molecules from an area of higher concentration to an area of lower concentration is known as
a. osmosis b. capillary action c. migration d. diffusion

21. Arteries
    a. carry blood to the heart
    b. are relatively thick-walled
    c. run between the heart and a capillary bed
    d. usually contain oxygenated blood

22. The advantages for large organisms of being made up of small cells include
    a. the ease of gas exchange
    b. the large surface to volume ratio
    c. the relative ease of diffusion of nutrients and waste molecules
    d. the ease of replacement of an individual cell

23. Nitrogen occurs in which class(es) of molecules?
    a. Carbohydrates   b. Proteins   c. Nucleic acids   d. Lipids

C. Matching.

[I] Match each example of interspecific interaction with the appropriate class(es). (4 pts)

examples -

24. bluebird and insects
25. finch and pine tree
26. bobcat & lion
27. bees & clover

classes

a. both species benefit (mutualism)
b. one species benefits and the other neither gains nor loses (commensalism)
c. one species benefits and the other is harmed (predation, parasitism, disease)
d. both species are harmed (competition)

[II] The circulatory system provides transportation for many substances in the body. Match each substance below with the cell, organ or gland from which it enters the bloodstream. (4 pts)

Transported Substance

28. insulin
29. glucose
30. nitrogenous wastes
31. blood cells

Cell, Organ or Gland

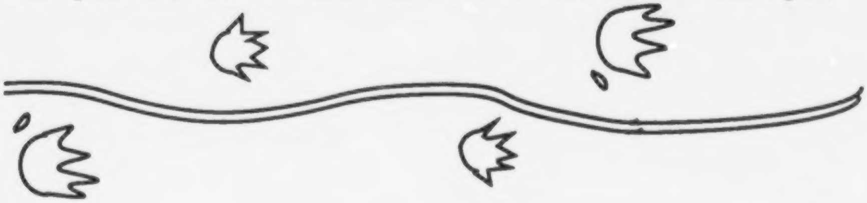
(a) long bones
(b) individual cells throughout body
(c) spleen
(d) liver
(e) pancreas

Name ________________

INITIAL SURVEY OF PROCESS SKILLS

Try to do what is asked to the best of your ability./ If you don't understand how to do something, explain what is puzzling you.

1. The following diagram shows tracings of a set of animal tracks. (Actual size.) Please make an observation based upon something you can see in the diagram and then draw a conclusion from your observation. I have given one possible observation and conclusion as an example.



Example:
Observation: There is a pair of lines running between the tracks.
Conclusion: Maybe this animal has a tail which is long enough that it drags when he walks.

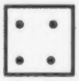
Your observation:

Your conclusion (based on the observation that you made):

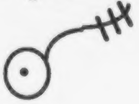
---

2. Pretend that you want to help a grade school child discover similarities and differences between the creatures at the bottom of the page. You decide to help him sort them by various characteristics. On the back of this page, explain what you would do. If you wish, you may make some kind of chart or diagram. You may find it easier to work with these creatures if you tear off the bottom of the page and either tape or redraw them on your diagram in the correct place.

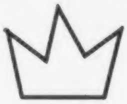
PICTURES
OF
"CREATURES"



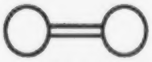
Nex



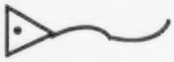
Jinx



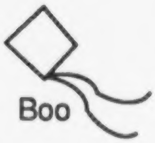
Poof



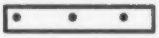
Tam



Wow



Boo



Hi

3. The students in a fifth grade class were asked to estimate the number of potato bugs in a garden which was 5 meters by 10 meters. They were only given 15 minutes, so they had to estimate the number in some way.

Team A decided to hunt for potato bugs in the bottom square meter. They found 10 sow bugs. Since one square meter is 1/50th of the total area, they estimated that there were 500 potato bugs in the garden.

Team B split up and each member counted potato bugs in a different square meter area. They found 10, 2, 3, and 6 potato bugs in their different areas. When they averaged these results, they got 5 potato bugs per square meter. They multiplied 5 bugs/square meter by 50 square meters total to get 250 potato bugs in the garden.

Which team's results do you think were more accurate, Team A's or Team B's?

Explain why you think so.

---

4. Which of the following tables is better? Explain your choice

Table A

Amount of Water (ml)	Number of Grass Seeds Germinated
0	0
20	32
40	40
60	35

Table B

Amount of Water	Number of Grass Seeds Germinated
20	32
60	35
40	40
0	0

5. Read the following two experiments.

Expt A: Jerri decided to find out which dry cat food cats prefer. She bought Whiskas, Friskies, and Science Diet. Each morning she provided her 3 cats with 10 oz of each cat food in separate bowls. At night she weighed the food remaining in each bowl. She repeated the experiment each day for a week. She found that most days her cats ate more Whiskas than Friskies or Science Diet.

Expt B: Toni decided to find out which dry cat food cats prefer. She bought Whiskas, Friskies, and Science Diet. On Monday, she provided her 3 cats with 10 oz of each cat food in separate bowls. That night she weighed the food remaining in each bowl. She found that her cats ate more Whiskas than Friskies or Science Diet.

In the above experiments, what three samples did the experimenter test?

What did he/she want to find out? What did he/she count or measure?

Explain why you can trust the results of Expt. A more than the results of Expt. B.

---

6. On the graph paper provided make a bar graph which shows the lengths of the various parts of the digestive tract.

PART	LENGTH	UNITS
Mouth	8	cm
Esophagus	43	cm
***	*	*
Small intestine	6.4	m
Large intestine	1.5	m

Note that the length of some of the parts is given in centimeters, while the length of other parts is given in meters.

APPENDIX C.3

Sample Lessons

NS412C

01. Is It Alive?
(Characteristics of Living Things)

A. The Problem

The problem we are studying today (What does it mean to be alive?) has been pondered for centuries. It recently was highlighted by Carl Sagan in the NASA space program. That is, we hope to find other forms of life in outer space, but how will we recognize life forms if they are totally unfamiliar? What criteria will we use to distinguish between living and non-living?

According to science education researcher Susan Carey, children between 5 and 7 are just beginning to form the concept 'alive.' Initially they distinguish between 'animate' and 'inanimate' rather than 'alive' and 'dead' or 'alive' and 'non-living.'

B. The Plan

You will answer the questions below in two successive cycles. Working independently, you will first form your own conclusions. Next, you will compare your conclusions with your group, and together you will generate the best synthesis of your ideas.

Results are to be carefully recorded in your notebook. Keep notes throughout the course, and add to them as new thoughts occur to you. Keeping good records is an important part of being 'scientific'.

C. Background Understanding

1. Thinking about living and non-living things you have known, and observing the living and non-living things in this room, make a list of attributes that are characteristic of most living things but not characteristic of most non-living things.

Characteristics of Living & Non-Living Things

Living	Non-Living

Describe what happens to each attribute of living things as a living thing ages and dies. Does the characteristic change with aging? Does it vanish before death? Does it persist after death?

Name one or more specimens that are on the border between living and non-living.

D. Classifying Unknowns & Reporting Results

Pretend you have just brought home your groceries. Do you put them anywhere in the kitchen or do you put bread, canned beans, and tuna fish in the cupboard; eggs, cheese, and strawberries in the refrigerator; and ice cream and frozen yogurt in the freezer? If you do the latter, you are classifying. As soon as children start comparing objects they begin to group them. As they develop, children learn to classify by multiple characteristics and to make and remake subgroups depending on their purpose. Scientists also use classification skills in many situations, as in organizing data into tables. In this lab, you are to classify certain 'unknowns' according to whether they are alive, dead, or never lived. You should also indicate why you chose the classification you did. Record your results in the form of a concise table similar to the example shown below.

Classification of Unknowns

	Specimen	Classification			Reasons
No.	Description	Alive	Dead	Never Lived	for Classification

Note that a good table has several parts::
- a clear, short, informative heading that tells the main point of the table
- clearly labeled columns and rows, organized in a logical fashion
- little or preferably no redundancy of information
- well-organized data

A good table provides a concise and comprehensible summary of results.

E. Summary Questions

1. Name three characteristics of living things that usually persist at least for a while after an organism dies.

2. Name three characteristics of living things that stop almost immediately upon death.

2. Name three key differences between dead specimens and things that never lived.

3. Define and distinguish between the terms 'organism and 'specimen'.

4. Define 'alive'.

5. Which of the following classes of organisms are alive? Animals, plants, fungi, one-celled protista, bacteria, viruses.

6. Describe some of the difficulties you foresee in recognizing life in outer space.

Compare your answers with your teammates.

Some possible answers: 1) general shape and form; overall organization including key organ systems; cellular composition; 2) respiration, metabolism; requirement for energy; 3) things that never lived typically have no cell structure; different chemical composition; less complex structure; 4) alive - to be living, characterized by energy consumption, metabolism, respiration, responsiveness, capability for reproduction, derived from living things, susceptible to dying); 5) all but viruses are alive; 6) ??

02. Mendelian Traits

Materials:
Graph Paper

A. Organizing Initial Knowledge

1. Name 4-5 of your physical characteristics that you believe to be inherited. Identify some of your blood relatives (for example, parent, sibling, grandparent,.aunt, uncle, cousin, child) who share the same characteristic.

Family Traits	
Characteristic	Family members who share characteristic (name and relation to you)

2. What does it mean for a trait or characteristic to be inherited?

B. Observation

(1) Classify yourself with respect to the following traits, by circling, for each trait, the first two pairs of symbols (homozygous dominant or heterozygous) or the third symbol (homozygous recessive) e.g., **LL** or **Ll** if dark hair or ll if blond.

Category	Symbols	Description
1. Hair color	LL or Ll ll	brown, black, or red hair blond hair
2. Hair type	TT or Tt tt	naturally curly naturally straight

3. Tongue curling	CC or Cc cc	can curl tongue cannot curl tongue
4. Mid-digital hair	MM or Mm mm	hair present, middle digit of finger hair absent, middle digit of finger
5. Pigmented iris	EE or Ee ee	eyes not blue blue eyes
6. Widow's peak	WW, Ww	peak in center of hairline
7. Bent finger	BB or Bb bb	little finger curves toward others little finger straight

There are a number of things to know about these traits.

a. Each of the traits above is produced by a single gene (whereas many other traits, such as skin color or weight, are affected by many genes).
b. Single gene traits are called Mendelian traits because they follow the simple inheritance patterns described by Gregor Mendell.
c. All humans have two copies of each gene, called alleles.
d. A dominant allele will be expressed whether it is homozygous (that is, occurs with another dominant allele of the same type, such as LL) or heterozygous (that is, occurs in combination with a recessive allele, such as Ll).
e. A recessive allele is expressed when it is paired with another recessive allele of the same type (such as ll), but it is masked when combined with a dominant all (such as **Ll**).
f. Most dominant traits look alike whether they are produced by homozygous dominant alleles (e.g., **LL**) or heterozygous alleles (e.g., **Ll**). Thus, if you have a widow's peak you don't know if your alleles are **WW** or **Ww**.

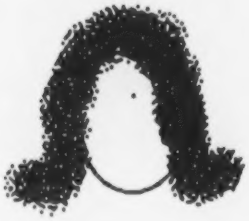
into the quarter wheel you have selected and fill in C_ or cc.....and so on. When you have completed the wheel, write the number of your type below:

C. Data Summary

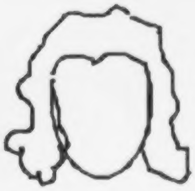
1. Summarize phenotypes of all students in the class in chart below.

Table 2.1: Number of Individuals of Each Type in Class
(raw data summary)
(put a slash mark under the number corresponding to the genetic type of each student)

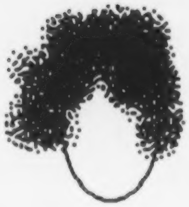
1	2	3	4	5	6	7	8	9	10	11	12
13	14	15	16	17	18	19	20	21	22	23	24
25	26	27	28	29	30	31	32	33	34	35	36
37	38	39	40	41	42	43	44	45	46	47	48
49	50	51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70	71	72
73	74	75	76	77	78	79	80	81	82	83	84
85	86	87	88	89	90	91	92	93	94	95	96
97	98	99	100	101	102	103	104	105	106	107	108
109	110	111	112	113	114	115	116	117	118	119	120
121	122	123	124	125	126	127	128				



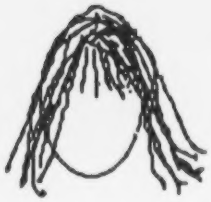
1. dark hair



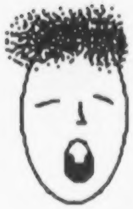
light hair



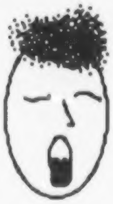
2. curly hair



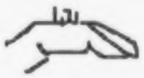
straight hair



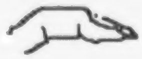
3. curls tongue



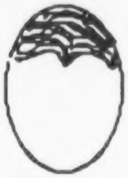
can't curl tongue



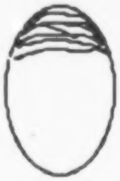
4. mid-digital hair present



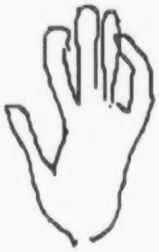
mid-digital hair absent



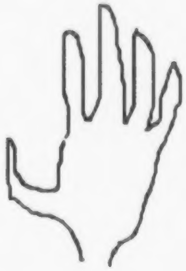
6. widow's peak



no peak



7. bent little finger



straight little finger

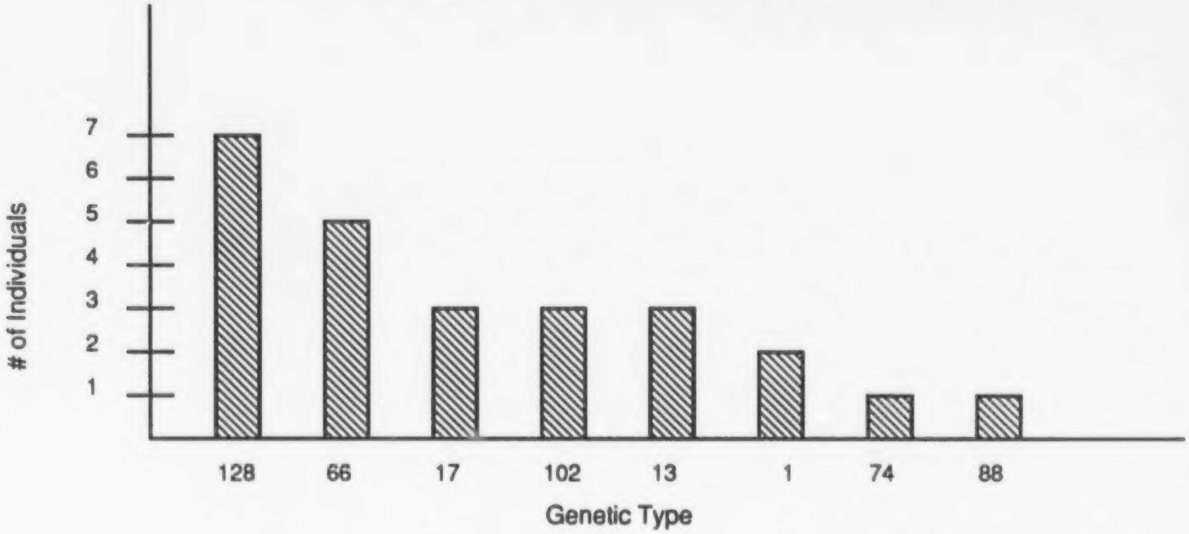
(2) If we consider the homozygous dominant (**LL**) and the heterozygote (**Ll**) to be indistinguishable, these seven pairs of alleles, each of which can produce two alternative characteristics or traits, can be combined in 128 different ways. That is,

$$2 * 2 * 2 * 2 * 2 * 2 * 2 = 128.$$

Determine which combination (1-128) represents you by mapping your type onto the genetic wheel. Begin in the center and color in **L**_ (which stands for either **LL** or **Ll**) or **ll**. Then move out into the half of the wheel you have selected and fill in **T**_ or **tt**. Then move out

2. Convert this summary of raw data to a polished graph such as the one shown below.

Table 2.3: Genetics Types of 25 NS 412 Students



3. Summarize the data in an alternative way, showing how many individuals have the dominant or recessive form of each trait, as shown.

Table 2.2. Number of Dominant and Recessive Phenotypes in Class (raw data summary)

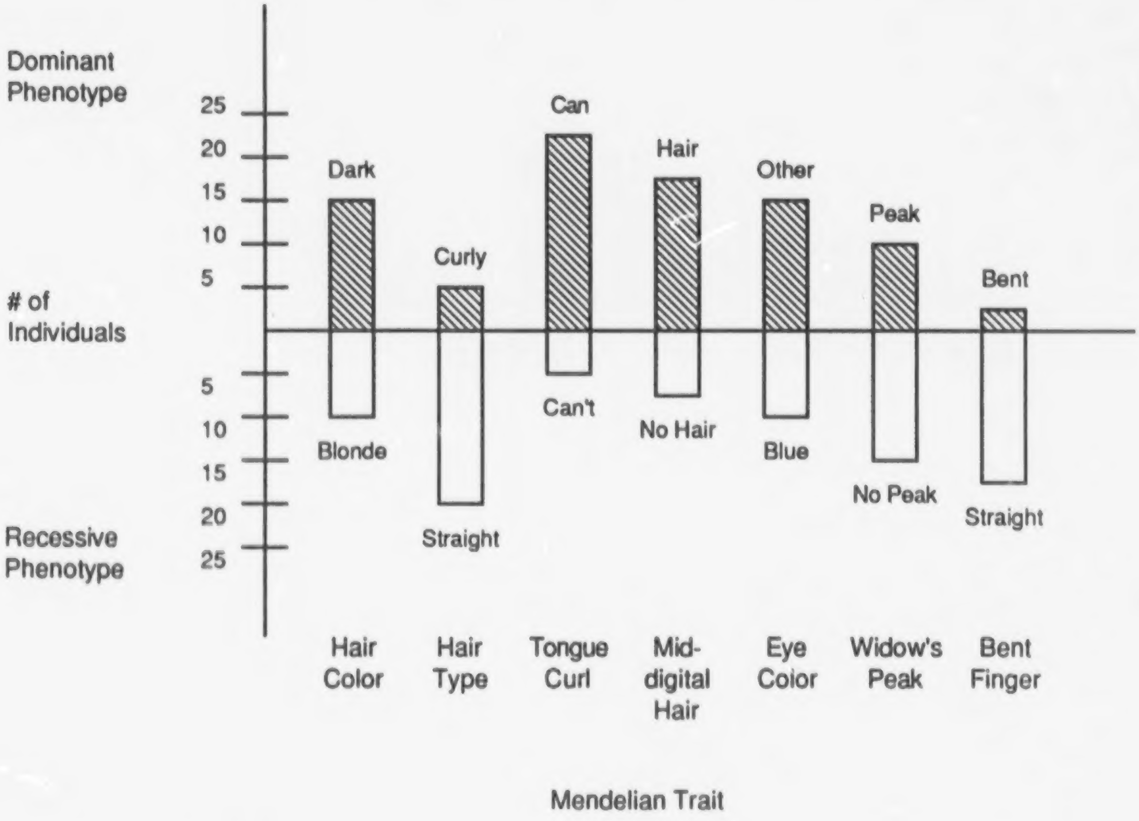
	Hair Color	Hair Type	Tongue Curl	Mid Dig Hair	Eye Color	Widow's Peak	Bent Finger
	Dark	Curly	Can Curl	Has Hair	Not Blue	Has Peak	Has Bent
Dominant Phenotype							
Recessive Phenotype							
	Blonde	Straight	Can't	No	Blue	No	Not

	Curl	Hair		Peak	Bent

re the previous table -
- put a hash mark in one of the two boxes (above or below the line) for each trait that corresponds to your phenotype;
- organize summary of hash marks representing class types in groups of five - i.e., ~~IIII~~)

4. Using graph paper, transform your raw data summary (above) into a bar graph that extends on both sides of the axis. Follow the example below.

Table 2.4: Frequency of Dominant & Recessive Phenotypes Among NS 412 Students



D. Questions & Comments

1. Notice that:
   a. Each bar graph has an informative title.

b. The horizontal and vertical axes are named.
c. The total number of individuals is 25, for each graph.
d. Each bar is identified.
e. The numerical scale is regular and numbered.

2. Some common misunderstandings -

Are all desirable genes dominant? [No - for example, the gene for 6 fingers is dominant to the gene for 5 fingers.]

Are all dominant genes desirable? [No. - see above example]

Is the dominant allele typically the most prevalent form of a gene in the population? [No, not necessarily. - the dominant gene for 6 fingers is less prevalent than the recessive gene for 5 fingers]

3. Here are some other traits of human beings that are produced by a single pair of alleles.

Mendelian Traits

Dominant	recessive
long ear lobes	short ear lobes
white skin spotting	uniform skin color
astigmatism	no astigmatism
normal body size	midget
6 or more fingers	5 fingers
varicose veins	no varicose veins

4. How would you go about designing a genetic wheel to summarize these characteristics?

5. If more genetic traits were included, would the number of possible phenotypes increase of decrease, relative to the 128 types we observed? Explain.

6. It has been estimated that humans have about 50,000 genes. Does this help you to understand why each person is different from every other person?

05. Quantitative Variation
(The Metric System)

Equipment:

balances	English & metric rulers
English weights	English & metric measures of volume
metric weights	English & metric scale (for humans)
plastic volume demos	

A. Introduction

Many of the observations we make are qualitative. The ball is red. The rock is hard. The boy is tall. John is heavier than Sam. Sometimes, however, qualitative observations are not sufficient. We want to compare to a standard. The boy is 4 feet tall. John weighs 5 pounds more than Sam. When we compare to a standard, we are making a quantitative observation, i.e. a measurement.

If you think for a moment, you will realize that someone initially had to define standards for length, volume, mass, and temperature. The English system, which we use in the United States, developed over a long period of time. As a result, no particular effort was made to make conversion from one unit to another convenient. You may, for example, have an intuitive idea as to how much fruit punch fits into a 5 gallon bowl, but could you say for sure that there would be enough punch for each of 50 wedding guests to have one cup each? Could you decide whether a 14 ounce package is more or less than a pound? Working with the English system requires a lot of experience. Thus, most children find the metric system, which is described below, much less frustrating.

The metric system is easier to remember and use because the scientists who developed it in France in 1791 based it on the decimal system. Thus, each unit is related to every other unit by some multiple of ten. (After the French Revolution, the new leaders didn't even want old units of measurement to remind them of the past!) Table 3-1 shows the names and abbreviations of the standard units in the metric system.

Table 3.1 Standard Metric Units

Property	Name of Unit	Abbreviation
	Defined Units	
Length	Meter	m
Mass	Gram	g
Temperature	Degree Centigrade	°C
	Derived Units	
Square Area	Meter Squared	m^2
Cubic Volume	Meter Cubed	m^3
Volume	Liter	l
Density	Gram/Milliliter*	g/m

*Milliliter will be defined below.

Units smaller or larger than the standard are made by adding suffixes. The most commonly used suffixes are milli- for 1/1000 of the standard unit and kilo- for 1000 times the standard unit. The suffix centi- is used occasionally. A centimeter, which is 1/100 of a meter, is an especially useful length since 2.5 cm is approximately equal to an inch.

Since both the metric and the English system are used in the United States, the author has found it useful to know a few approximate conversions. Notice the emphasis on few and approximate. You would hardly bring along a book of formulae to the grocery store and I doubt you would stand by the wine counter with a calculator! However, you might find it useful to know that a cup is about 250 ml and thus the liter of wine which you are thinking about buying will serve at most 4 or 5 people.

The first step toward feeling comfortable with the two systems is to memorize the 10 most useful conversions listed in Table 3.2 and the temperatures in Table 3.3. Then it is just a matter of practice until you will be able to look at a jar and feel pretty sure that it will hold about a liter or look at a davenport and guess that is it about 2 meters long.

Table 3.2 Useful Metric/English Conversions

Volume

16 milliliters ≈ 1 tablespoon
250 milliliters ≈ 1 cup
1 liter < 1 quart
5 liters ≈ 1 gallon

Length

2.5 centimeters ≈ 1 inch
1 meter > 1 yard
1 kilometer ≈ 0.6 mile

Mass

5 grams ≈ 1 nickel
28 grams ≈ 1 ounce
454 grams ≈ 1 pound
1 kilogram ≈ 2.2 pounds

Note: milli = 1/1000th
centi = 1/100th
kilo = 1000x

Table 3.3 Temperature Conversions

Characteristic Temperature	Centigrade (degrees)	Fahrenheit (degrees)
Freezing water	0	32
Room Temperature	22	72
Warm Room*	28	82
Mammal's Body Temperature	37	98.6
Boiling Water	100	212

B. Challenges

1. What is the metric system of measurement? (definition/description)

2. Arrange the following quantities from largest to smallest.
   1 meter, 1 centimeter, 1 millimeter, 1 kilometer

3. Are the above units of weight or units of distance?

4. What is the approximate equivalent of each of these quantities in the American system of measure?

1 millimeter =
1 centimeter =
1 meter =
1 kilometer =

5. What is the approximate metric equivalent for each of the following?

1 pound =
1 cup =
1 quart =

6. Sort these English system terms into units of weight, units of volume, and units of length, from smallest to largest: **cup, fluid ounce, foot, gallon, inch, mile, ounce, pint, pound, quart, ton, yard**.

units of weight	units of volume	units of length

7. Sort these metric terms into units of weight, units of volume, and units of length, from smallest to largest: **centimeter, gram, kilogram, kilometer, liter, meter, microgram, micrometer, milligram, milliliter, millimeter**.

units of weight	units of volume	units of length

8. 15 meters equals how many centimeters? ___________

9. 1500 meters equals how many kilometers? ______________

10. A kilometer is six-tenths (slightly more than half) a _________

11. A kilogram is approximately equal to (a) 1 pound, (b) 2.2 pounds, (c) 10 pounds.

12. Two and one-half centimeters are approximately equal to a. an inch, b. a foot, c. a yard.

13. One meter is approximately equal to a __________ .

14. A liter is slightly smaller than a __________ .

15. Five grams is approximately equal to a (a) dime, (b) nickel, (c) quarter, (d) half dollar.

16. Measure 30 ml, 300 ml, and 10 ml. (see discussion about measuring liquids in Process Skills at back of syllabus.

Answers

1. What is the metric system of measurement?

2. Arrange the following quantities from largest to smallest.

1 meter, 1 centimeter, 1 millimeter, 1 kilometer

mm < cm < m < km

3. Are the above units of weight or units of distance? *distance*

4. What is the approximate equivalent of each of these quantities in the American system of measure?

1 centimeter = 0.4 inches
1 millimeter = 0.04 inches
1 kilometer ≅ 0.6 mile
1 meter = 1 yard

5. What is the approximate metric equivalent for each of the following?

1 pound = 454 gm
1 cup = one-fourth liter (0.24)
1 quart = 0.95 liter

6. Sort these English system terms into units of weight, units of volume, and units of length: cup, fluid ounce, foot, gallon, inch, mile, ounce, pint, pound, quart, ton, yard.

weight	volume	length
ounce	fluid ounce	inch
pound	cup	foot
ton	pint	yard
	quart	mile
	gallon	

7. Sort these metric terms into units of weight, units of volume, and units of length, from smallest to largest: centimeter, gram, kilogram, kilometer, liter, meter, microgram, micrometer, milligram, milliliter, millimeter.

weight	volume	length
microgram	milliliter	micrometer
milligram	liter	millimeter

gram
kilogram

centimeter
meter
kilometer

8. 15 meters equals how many centimeters? ___1500_

9. 1500 meters equals how many kilometers? __15__

Common English-Metric Conversions

10. A kilometer is six-tenths (slightly more than half) a __*mile*___

11. A kilogram is approximately equal to (a) 1 pound, (b) 2.2 pounds, (c) 10 pounds (pick one).

12. Two and one-half centimeters are approximately equal to a. an inch, b. a foot, c. a yard (pick one).

13. One meter is approximately equal to a _*yard* .

14. A liter is slightly smaller than a _*quart* .

15. Five grams is approximately equal to a (a) dime, (b) nickel, (c) quarter, (d) half dollar (pick one).

weight:	**volume**:	**length**:
1000 ug = 1 mg	1000 ml = l	1000 um = 1 mm
1000 mg = 1 gm		10 mm = 1 cm
1000 gm = 1 kg		100 cm = 1 m
100 cm		1000 m = 1 km

09. Beans, Seeds, & Plants

Bring to Class

half gallon milk or juice bottle (plasticized cardboard)
seed or bean of some type

Materials for Planting:

seeds
potting soil
milk cartons
water
3 beans

Materials for Seed (Bean) Germination

ziploc baggy
2 paper towels
water
masking tape
cardboard
6 beans
elastic band

Tools: Seed Germination

Scissors
Stapler
Spray bottle, water

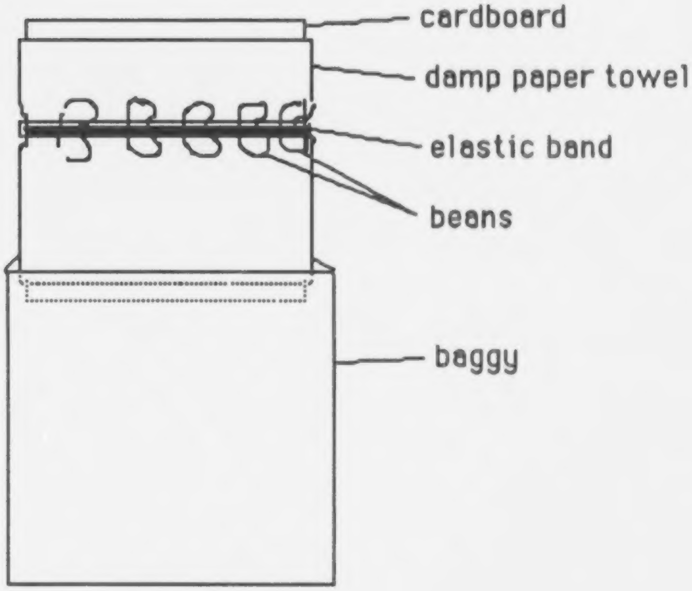
All higher organisms (that is, all living things except bacteria) go through a series of developmental stages, and the entire pattern is described as the organism's life cycle. If you have raised or are raising children, you have observed many developmental stages of humans. Have you ever planted a seed and watched it grow?

Seeds normally grow in the dark, since they typically are planted in the ground. Do you think that seeds require darkness to grow? Do you think they require soil to grow? Write down your predictions below, and then let's find out.

For each baggy,

a. Soak seeds (beans) in water for two-three three hours.
b. Thoroughly dampen two paper towels, but make sure they are not dripping wet. (It is important that they don't dry out, but if they are too wet, they will mold.)
c. Crush one of the damp paper towels and place in the bottom of the baggy.
d. Cut a piece of cardboard so it just fits inside a baggy, allowing you to seal the top.

Prepare a baggy (seed germination chamber) as follows.



e. Lay the second damp paper towel on the cardboard and fold it around the cardboard.
f. Place an elastic band around the cardboard and towel about 1/3 of way from top.
g. Place 5-6 beans under the elastic band and perpendicular to it, all on one side of cardboard.
h. Place the cardboard in the baggy.
i. Staple the baggy and cardboard underneath the beans in two or three places to help hold the beans in place.
i. Seal the baggy about half way across to conserve moisture, but leave partly open to permit some air circulation.
j. Use masking tape to label the baggy with your name and the date.

Tape the baggy where it receives good light from the window. Check the baggy each lab period to make certain that it remains moist and free of fungus. Use a spray bottle to add moisture if it dries out. Water between lab periods if necessary.

Keep a record of the changes you observe in each group of seeds, or if no changes occur. The record should be chronological (that is, organized by date) as in the following example:

9/4 Seeds that had been soaked in water for four hours were placed in a vertical position in two 'seed germination chambers' (baggies) on a paper towel that *was moist but not wet; they were held in place with an elastic band and staples below the seeds; one baggy was hung in the light and the other was hung near it but was wrapped in aluminum foil to prevent any light from entering*

9/6 No change in seeds

9/11 Signs of growth in both baggies; small projections from indented side of seeds # 3 & 5

9/13 etc.....

Here are some questions you might be asking. Perhaps you can think of other questions as well.

a. Does light or darkness inhibit growth of seeds?

b. When a bean grows, what appears first, the root or the stem?

c. Do the root and stem 'know' what direction to grow in? If so, how?

d. Where does the young seedling get the energy for growth?

e. Can a seed grow indefinitely with only water supplied to it?

f. What conditions trigger bean seed germination?

g. What do we mean by 'germination'?

h. How does the growth of a plant seed such as a bean compare to the growth of a human 'seed' - that is, a fertilized egg?

Plant three kidney bean seeds that have been soaked in water, and the bean or seed(s) you brought from home, in a half-gallon container (the larger the container, the less often it needs to be watered).

1. Put holes in the bottom of the container for drainage.
2. Fill the container to about 3/4 inch from the top with potting soil.
3. Tamp the soil down well to eliminate air spaces.
4. Place the seeds about one half-inch below the soil.
5. Water thoroughly and allow to drain.
6. Write your name, date, and the name of the seeds on the container.
7. Place the container on a tray to go under the growth lights.

10. Cricket & Mealworm Life Cycles

(adapted from Dr. Phoebe Roeder)

Bring to Class

One plastic soda bottle (quart or liter) with lid, per group
3-4 leaves

Mealworm Materials:

container of mealworms from a pet store
bran or bran flakes
plastic dish
slice of apple
paper towel

Cricket Materials:

large waste basket containing adult crickets
(The pet store on University at about 30th will let you pick adults.)
pop bottles or terraria
potting soil
egg cartons for houses
alcohol burner
sharp knife or razor
dry dog food
small jar to keep dog food dry
nail

Purposes

To observe and draw pictures of the body structure of adult crickets and mealworm larvae.

To compare the developmental stages of crickets to those of mealworms.

References

1. Crickets	SCIS, Grade 2
2. Mealworms	SCIS, Grade 2
	Silver Burdett Science, Grade 3, p. 12.
3. Butterfly	Silver Burdett Science, Grade K, p. 52.
4. Terraria	Silver Burdett Science, Grade 2, p.49

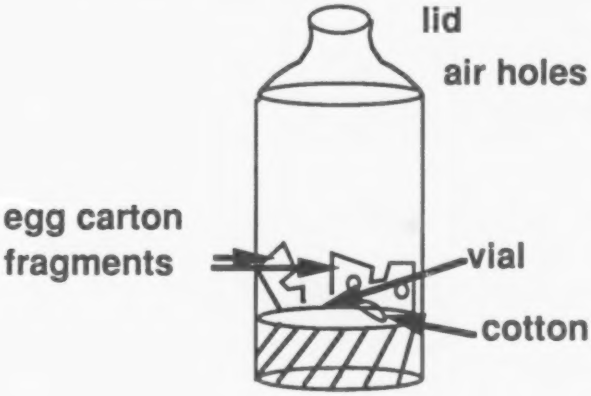
Procedure for Setting Up Mealworm Containers:

1. Fill a tray with 1 cm of bran flakes
   (A baggy can also be used for mealworms.)
2. Cover the flakes with a folded paper towel
3. Place a slice of apple on top of the towel
4. Add about 10 mealworms

Procedure for Setting Up Cricket Terrarium

1. Heat a nail in a flame and use it to melt 4 air holes in the curved top of the plastic quart-size soda bottle.

2. Use a very sharp knife or razor blade to partially cut off the top of the soda bottle at the point between the straight sides and the curved top. Leave a small section uncut as a hinge.
3. Mix potting soil with water so that it is lightly moist but not wet. Pack soil into the bottom of the jar. It should come at least 1 cm above the opaque bottom so you will be able to see eggs in the dirt.
4. Add 3-4 leaves and a piece of egg carton as hiding places.
5. Put a small amount of cornmeal or other ground grain in a 50 ml vial. Place the vial on its side in the cage so crickets can crawl in and get food.
6. Place a piece of cotton or handiwipe soaked in water on the soil.
7. Capture 6 crickets in a tall cup (from the supply in the waste basket) and add them to your terrarium. Be careful to avoid damaging their limbs as you capture and move them.
8. Tape the top of the bottle closed.



Maintenance

1. Add apple to mealworms as needed and change to a fresh paper towel at the same time
2. Add moisture to cotton ball or handiwipe if needed (it probably won't be required unless the room temperature is quite hot)

Observations & Records

Day 1 (3/12) -

1. Observe organisms closely.
    a. Note the number of body segments, the number of appendages, and the shape of each appendage and where it connects to the body (more questions below). Draw carefully and with precision.

b. Note how each requirement for life is satisfied (that is, how does the organism eat, how does it breathe, how does it eliminate wastes, how does it reproduce, etc.)

c. Observe the difference between males and females (look for ovipositor).

2. Observe the dirt in the cricket terrarium carefully so you will be able to detect eggs when they are deposited.
3. Have a precise count of the number of organisms in your container.

Day 2, 3/14) - Casual observations during lab. Place containers on desks and observe while doing other things. Note any interesting observations and key changes in your notebook. (Some possible questions are: Are they cannibals? Do they make noises by rubbing their wings? Does the male and/or female chirp? How are eggs deposited?). Make a careful count: Note any changes in population size.

Day 3 (3/19) - Detailed drawings (15 minutes), noting any differences in size, shape, and population size from day 1.

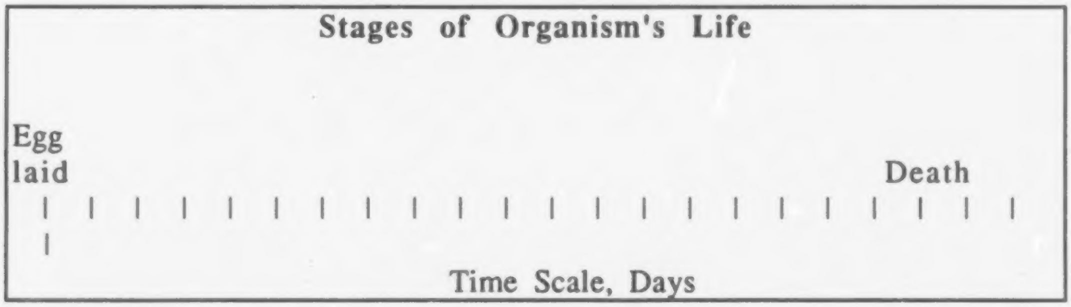
Day 4 (3/21) - Casual observations, observation of changes, & careful count like Day 2. Assure that specimens have good food and water supply for spring break.

Day 7 (4/2) and Day 8 (4/4) - Casual observations, observation of key changes, & careful count like Day 2.

Day 9 (4/9) and Day 10 (4/11) - Summary & Conclusions. Draw timeline, noting number of days required for each stage observed.

Predictions & Records

In your notebook, you should draw a timeline for both the cricket and mealworm. You should decide how long you think their life cycles are (prediction) and then draw an appropriate time scale under each timeline. Above each timeline, draw what you think these insects will look like at various stages of their lives.



Create a similar timeline to summarize your observations. Imagine that you are the first scientist ever to study the cricket and mealworm, so you must get your answers from the organisms themselves, not from a book.

Suggestions and Questions

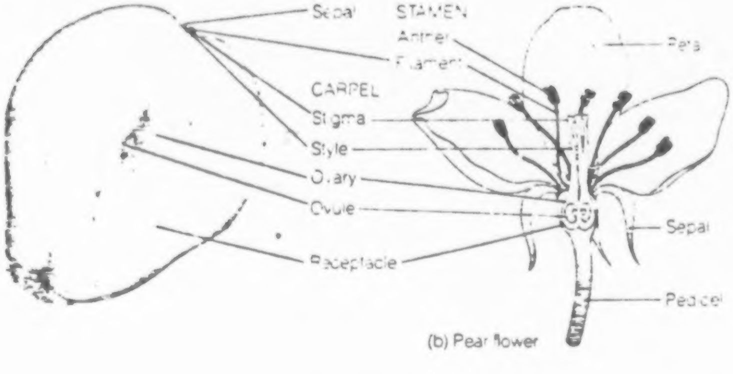
The following suggestions and questions may help you focus on the key ideas you should discover by the end of our 4 week observation period:

1. If possible, make observations on a dead cricket since it doesn't move constantly. If you don't have a dead cricket, you may want to put a live cricket into a small vial. You can leave the cap on the vial for about 5 min. without harming the cricket.
2. Notice where the legs attach, the size of the antennae compared to the body, the number of points on the rear, location and size of wings, number of body parts, etc.
3. Make the same observations for mealworms and later for mealworm adults.
4. How many males and females do you think you have? How could you test to be sure?
5. Realize that you can examine your crickets and mealworms only briefly during subsequent labs. (You can examine them before or after class for longer time periods if you wish).
6. Note the environmental factors that might affect survival of your insects.
7. Each day, ask yourself: Have any new forms appeared? Have any died? Can you find any eggs? Many or a few? How could you prove they are eggs?
8. When you first notice the appearance of baby crickets, ask yourself: In what ways do they resemble the adults? In what ways are they different? Are they all the same sex? How long did it take for them to hatch?
9. About how many babies were produced by the 6 crickets in your cage?
10. How many babies were produced in the most prolific culture?

11. Flower Structure

Materials

flowers
cutting surface
sharp cutting tool



(b) Pear flower

Method & Observations

a. A flower is a specialized structure for sexual reproduction. A "perfect" flower, containing both male and female parts, occurs in many plants. Some flowers, on the other hand, have only male or female parts.

b. There are typically sepals on the outside of the flower, often green, sometimes very small, sometimes as large as the petals. The sepals protect the bud before it opens.

c. The petals are often brightly colored to attract a pollinator, usually an insect or small bird..

d. The stamens are inside the petals; they are the male parts of the flower. Stamens may be T-shaped, colored, and straight or gently curved. Cut the flower open to see the stamens clearly. How many stamens do you see?

e. The stamens each have an anther at the top of the shaft. Pollen grains (haploid reproductive cells in a protective covering) are released from the anther. Can you see any pollen?

f. The carpel is the female part of the plant. There is usually one carpel per flower, in the center (but some flowers have more than one). To see the carpel clearly, gently separate the flower from the green sepals and base. The stamens will usually stay with the flower and the carpel remains attached to the base. This separation occurs naturally when a tree or plant sheds its flowers. The carpel has three parts: a sticky stigma at the top, a straight shaft called a style and an ovary at the bottom. Remove the sepals to see the ovary. The ovary contains the haploid egg.

g. Draw a picture of your flower(s) and label their parts.

h. When the plant sheds its flower, the fertilized egg (a 2N or diploid zygote) develops into a seed (with a protective coat and a nourishing endosperm), and the ovary develops into a fruit.

12. Organic Molecules & Energy
(Pattern Matching)

Materials

Cutout molecules
Control Materials: toast (will burn), sand (won't burn)
Test Materials: water, cooking oil, starch, sugar, gelatin (protein)

Equipment

burner matches tweezers aluminum foil

Part I

Studying science is in many ways like piecing together a puzzle. For this reason, **pattern matching** is an important skill. Biologists are constantly looking for significant patterns in nature. Even biological molecules are pattern matchers. Biological enzymes are a good example - each enzyme typically recognizes one and only one type of molecule.

In this lab you will exercise your pattern matching skills. You will aim for a higher level of recognition than the enzyme - that is, you will aim to perceive classes of objects rather than one single kind of object. The objects you will be sorting are the basic building blocks (subunits) of the common macromolecules in living things.

There are four pages of molecules altogether. Cut each page along the dividing lines to produce 12 cards from each, or a total of 48 cards. Discard the two blank cards. Each of the 46 remaining cards has one molecule displayed on it.

Organize the 46 cards into groups based upon structural similarities. Pay special attention to what **elements** (CHNOPS) are contained in the molecule, to the **shape** of the molecule, and to one or both ends of the molecule where there is often a **common structure**.

Common Elements in Living Things (CHNOPS)

Abbreviation	Name	Atomic Number*	Atomic Weight**	Usual # of Bonds+
C	carbon	6	12	4
H	hydrogen	1	1	1
N	nitrogen	7	14	3
O	oxygen	8	16	2
P	phosphorus	15	30	5
S	sulfur	16	32	2

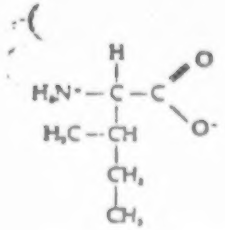
* the atomic number tells the number of electrons, protons, or neutrons in atoms; for example, oxygen has eight electrons, eight

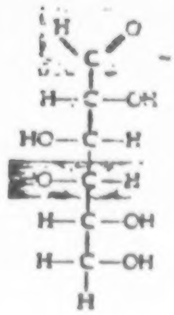
protons, and eight neutrons; all 'standard' forms of atoms except hydrogen have equal numbers of the three types of subatomic particles (hydrogen, the exception, has one proton and one electron but no neutrons)

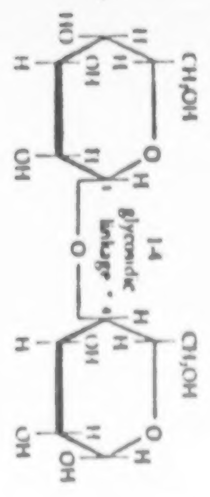
** the atomic weight is the sum of the protons and neutrons in an atom; each of these subatomic particles has a weight of one

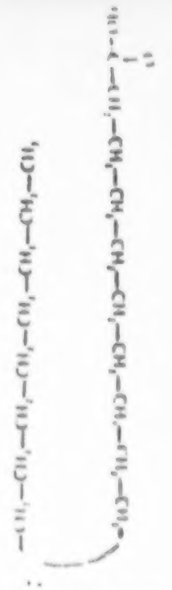
+ each atom can bond with other atoms; this column tells the usual number of bonds that are formed; for example, phosphorus often forms a single bond with each of three hydroxyl molecules (-OH) and a double bond with oxygen, or five bonds altogether

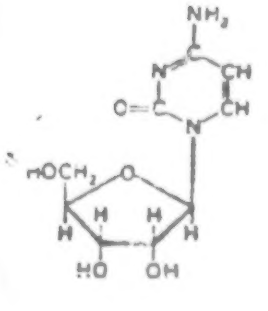
See if you can identify 7 major classes. Do NOT refer to your books during this exercise. Use your own judgement in doing the sorting. Finally, compare your categories with those of the other groups, and specify why you grouped molecules together as you did. That is, what are the **salient features** of each group of molecules?

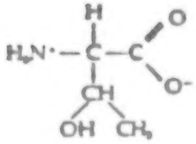

H
H₂N⁺
O
O⁻
H₃C
CH
CH₂
CH₃

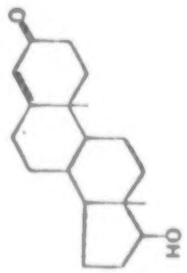

H
O
OH
HO
H

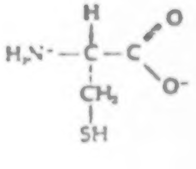

CH₂OH
1-4
glycosidic
linkage
OH
HO
O

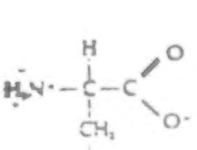


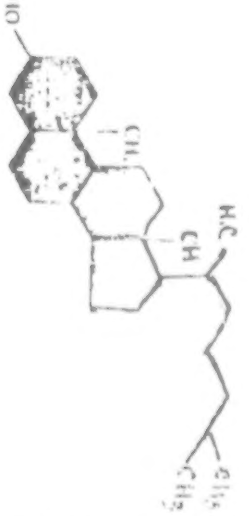

NH₂
N
CH
O
N
HOCH₂
H
HO
OH

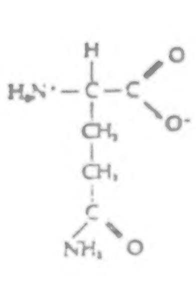

H
H₂N⁺
O
O⁻
CH
OH
CH₃

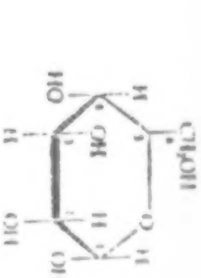

O
OH

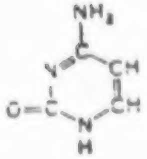

H
H₂N⁺
O
O⁻
CH₂
SH

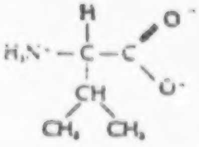

H
H₂N⁺
O
O⁻
CH₂
OH

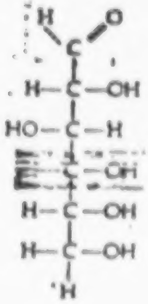

HO
CH₃
CH
H₃C
CH₃
CH₃

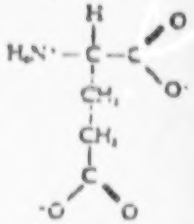

H
H₂N⁺
O
O⁻
CH₂
CH₂
C
NH₂
O

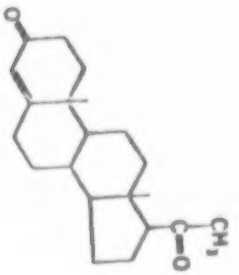

CH₂OH
OH
HO
O

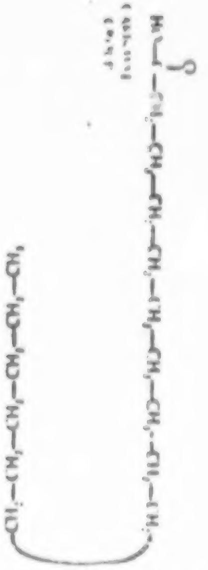

NH2
N
C
CH
O=C
N
H

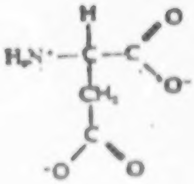

H
H3N+
C
O
O-
CH
CH3
CH3

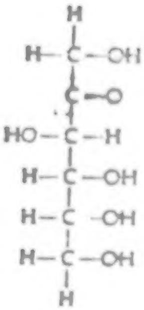

H
O
H
OH
HO
H
OH
H
OH
H
OH
H

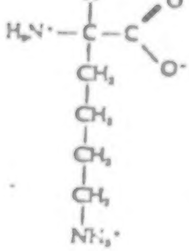

H
H3N+
O
O-
CH2
CH2
-O
O

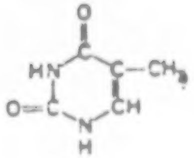

O
CH3
O

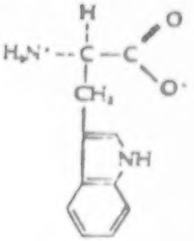


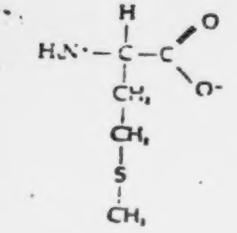

H
H3N+
O
O-
CH2
-O
O

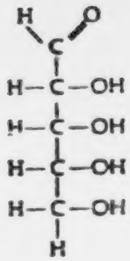

H
H
OH
C=O
HO
H
H
OH
H
OH
H
OH
H

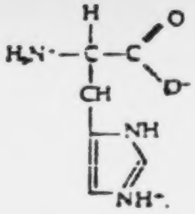

H
H3N+
O
O-
CH2
CH2
CH2
CH2
NH3+

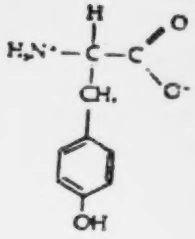

O
HN
C
C—CH3
O=C
CH
N
H

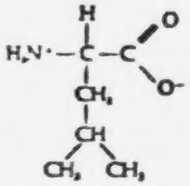

H
H3N+
O
O-
CH2
NH

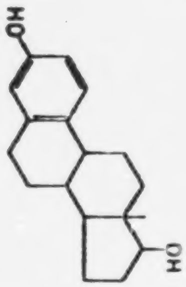


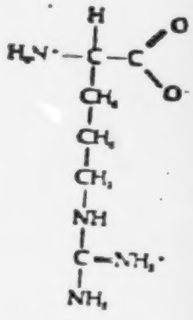


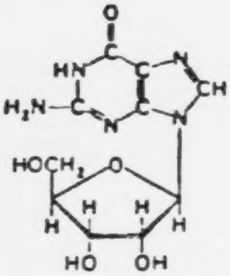


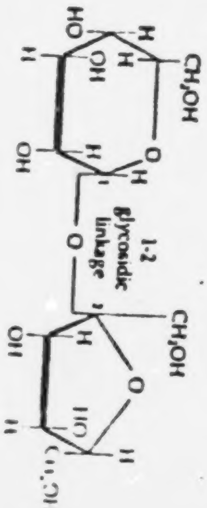


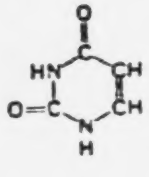


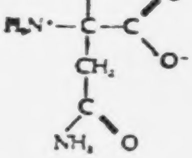


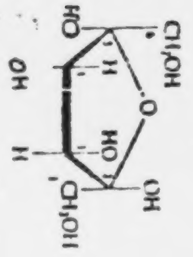


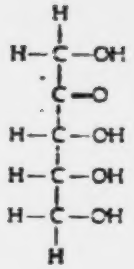


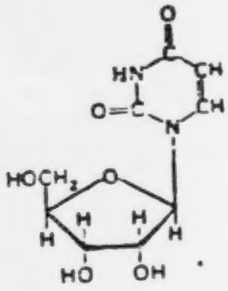

1:2 glycosidic linkage

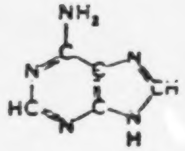


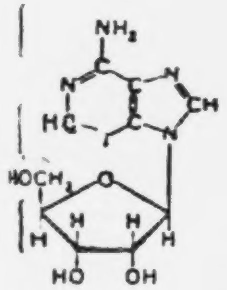


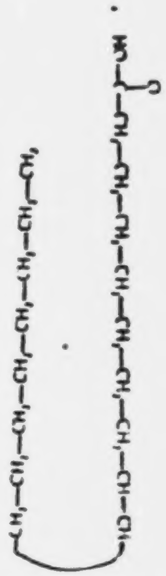


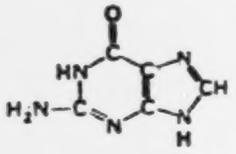


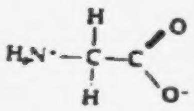


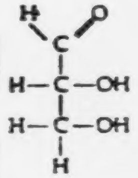


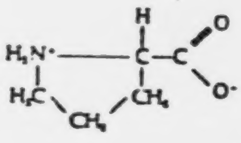


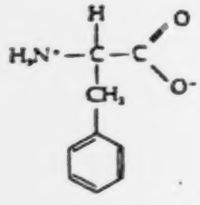


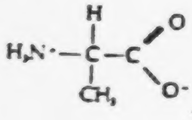












PERIODIC TABLE OF THE ELEMENTS

Period	IA	IIA	IIIB	IVB	VB	VIB	VIIB	VIII			IB	IIB	IIIA	IVA	VA	VIA	VIIA	0
1	1.008 H 1																	4.003 He 2
2	6.941 Li 3	9.012 Be 4											10.81 B 5	12.01 C 6	14.01 N 7	16.00 O 8	19.00 F 9	20.18 Ne 10
3	22.99 Na 11	24.31 Mg 12											26.98 Al 13	28.09 Si 14	30.97 P 15	32.06 S 16	35.45 Cl 17	39.95 Ar 18
4	39.10 K 19	40.08 Ca 20	44.96 Sc 21	47.90 Ti 22	50.94 V 23	52.00 Cr 24	54.94 Mn 25	55.85 Fe 26	58.93 Co 27	58.71 Ni 28	63.54 Cu 29	65.37 Zn 30	69.72 Ga 31	72.59 Ge 32	74.92 As 33	78.96 Se 34	79.90 Br 35	83.80 Kr 36
5	85.47 Rb 37	87.62 Sr 38	88.91 Y 39	91.22 Zr 40	92.91 Nb 41	95.94 Mo 42	[97] Tc 43	101.1 Ru 44	102.9 Rh 45	106.4 Pd 46	107.9 Ag 47	112.4 Cd 48	114.8 In 49	118.7 Sn 50	121.8 Sb 51	127.6 Te 52	126.9 I 53	131.3 Xe 54
6	132.9 Cs 55	137.3 Ba 56	• 57-71	178.5 Hf 72	180.9 Ta 73	183.9 W 74	186.2 Re 75	190.2 Os 76	192.2 Ir 77	195.1 Pt 78	197.0 Au 79	200.6 Hg 80	204.4 Tl 81	207.2 Pb 82	209.0 Bi 83	[209] Po 84	[210] At 85	[222] Rn 86
7	[223] Fr 87	226.0 Ra 88	† 89-103	[261] Rf 104	[262] Ha 105	[263] 106												

•	138.9 La 57	140.1 Ce 58	140.9 Pr 59	144.2 Nd 60	[145] Pm 61	150.4 Sm 62	152.0 Eu 63	157.3 Gd 64	158.9 Tb 65	162.5 Dy 66	164.9 Ho 67	167.3 Er 68	168.9 Tm 69	173.0 Yb 70	175.0 Lu 71
†	[227] Ac 89	232.0 Th 90	231.0 Pa 91	238.0 U 92	237.0 Np 93	[244] Pu 94	[243] Am 95	[247] Cm 96	[247] Bk 97	[251] Cf 98	[254] Es 99	[257] Fm 100	[258] Md 101	[255] No 102	[256] Lr 103

Part II

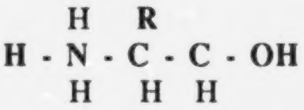
Identify the various kinds of molecules.

a. The general formula for sugars is $(C_6H_{12}O_6)_n$. That is, sugars contain carbon and oxygen in equal amounts, with twice as much hydrogen. The molecules may be ring structures or straight chains. Put all the sugars together. Divide them into three subgroups according to the number of carbons.

b. Nucleic acids contain nitrogen and phosphate groups ($PO_4^{=}$) as well as CHO. They may have one six-membered ring; this class is called the *pyrimidines* and includes cytosine, thymine, and uracil. Another class, called *purines*, has one 5- and one 6-membered ring; the common purines are guanine and adenine. Refer to the book to pair the nucleic acids as they occur in DNA and RNA (A=T, C=G, A=U). Which one is missing its mate?

c. Lipids contain many carbon and hydrogen atoms, with relatively little oxygen. The basic building block of lipids is the fatty acid. Phospholipids have a phosphate group at one end. Phospholipids are amphipathic molecules; they have a charged, water-loving phosphate group at one end, and uncharged, non-polar, water-hating fatty acid chains at the other end. Other lipids include steroids (ring structures such as estrogen, testosterone, and cholesterol) and waxes.

d. Proteins are made up of 20 common amino acids. Each amino acid contains nitrogen as well as CHO and has a backbone like this:



$$\begin{array}{ccccccccc} & & H & & R & & & & \\ H & - & N & - & C & - & C & - & OH \\ & & H & & H & & H & & \end{array}$$

where R stands for a side group that is different for each.

Part III

Organic molecules contain energy. The energy is stored in the covalent bonds that connect atoms together in each molecule. This energy is released when the bonds are broken. Living things have batteries of enzymes to help them to break the bonds in organic molecules and capture the energy for use in synthesizing their own molecules.

Which class of molecules contains the most energy per gram? (Hint: Which class of molecules is used to power jet and automobile engines? Which would form the hottest fire on the stove?) ______

If a substance does not burn, it does not contain the kind of chemical energy that is useful to living things. If it does burn, some but not necessarily all living things can probably use the substance as a source of energy. In this experiment, you will determine if the classes of organic molecules you just identified can burn, and you will compare them to a common inorganic molecule, water. (Note: organic molecules contain carbon).

Experiment. Place a small amount of each substance listed under materials (except for molecule cutouts) on a small piece of aluminum foil and, using tweezers, hold it over a flame to see if it burns. Bend the corners of the foil up so it makes a little cup. Use a small sample (less than 2 mm across).

The chemical bond between carbon and hydrogen contain the most energy. The bond between hydrogen and oxygen contain the least energy. Do you observations support these statements?

Summarize your results in the form of a table.

13. Water
(The Ubiquitous Molecule)

Equipment

hot plates
stirrers
scissors
beaker tongs
test tubes
10 ml grad cylinders
beakers - large & small
medicine droppers
thermometers
knives
glass slides
50 ml graduated cylinders

Supplies

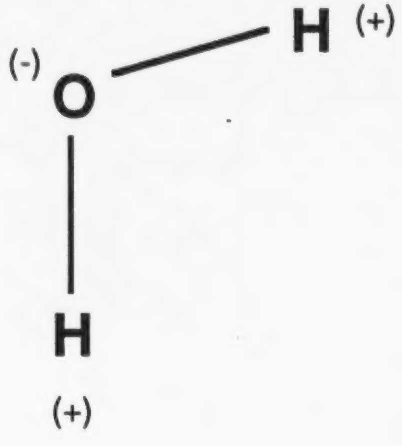
paraffin
cooking oil
paper strips
water
red dye
ink #1
wax paper
ice

Organizing Initial Knowledge

We are what we eat and drink, with a little supervision from our genes. In fact, we (living things) are mostly water. Thus, the simple water molecule can tell us a great deal about ourselves and the nature of life.

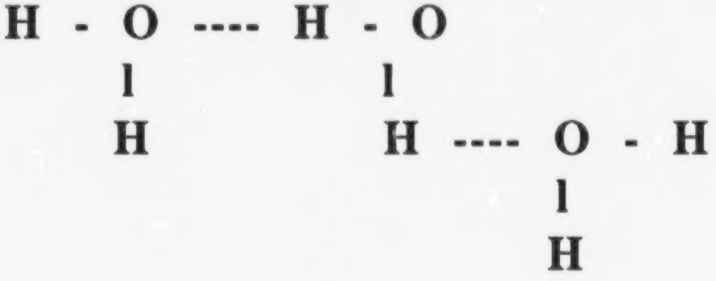
How do we study water and other molecules that are too small to see? In this lab we'll find a variety of ways.

1. Each molecule of water is made up of two atoms of hydrogen connected to one atom of oxygen, as shown below. This is summarized in the familiar formula, H_2O. The bonds are covalent, meaning that the linked atoms share a pair of electrons.



The oxygen atom is eight times larger than the hydrogen atom, and because of this, it holds the shared electrons about eight times 'tighter'. For this reason, and because each electron has a negative charge, the oxygen atom in each water molecule has a partial negative charge and the hydrogen atom has a partial positive charge.

2. Because of these partial charges, water molecules 'stick' to each other like a pair of magnets. The 'stickiness' is due to 'hydrogen bonding', the attraction of the hydrogen atom of one water molecule to the oxygen atom in another water



3. How does the 'stickiness' of water help account for the ability of water to rise from the roots of a redwood tree to the very top? (Hint: the cells of the tree contain a lot of water).

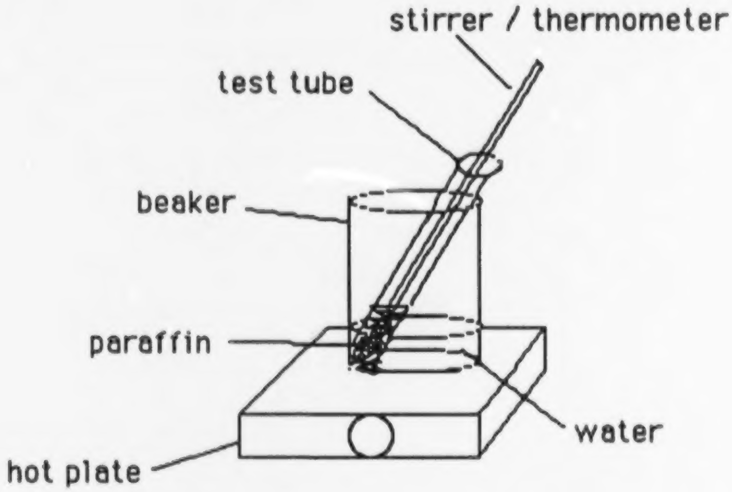
4. How does the stickiness of water make it possible for insects to walk on water?

Experiments

A. Density & Melting Point

1. **Paraffin** (wax). Melt approximately 30 grams of paraffin in a test tube, standing in a beaker of water on a hot plate. Stir as you measure

to keep it mixed very well. Record the temperature at which the paraffin just starts to melt (the melting point). ____________ degrees __ (C or F)



If you were to add solid chunks of paraffin to the melted paraffin, would the chunks float or sink? (what do you predict?)

____

Now try the experiment. Add solid chunks of paraffin to the melted paraffin, and push the solid chunks below the surface to break the surface tension. Do they bob back up, like ice cubes, or sink to the bottom?

____

What are your conclusions? that is, does paraffin differ in density in its solid and liquid states? _________

Which is more dense, solid or liquid paraffin? ____________________

[When you are done, let the paraffin harden again in the **standing** test tube]

2. **Water**. Fill a beaker about half full with about 2/3 chopped ice and one-third water. Stir vigorously but do not break. Measure and record the temperature of the mixture (the melting point of ice, the freezing point of water)

Record the temperature immediately after the last piece of ice has melted. If you add some ice cubes to the water now, will the ice cubes float or sink? (Predict!) ____________

Do the experiment (that is, add the ice cubes) and see if your prediction was correct. Does water differ in density in its solid and liquid states? _________ How?

Does water differ from paraffin with respect to its solid and liquid states? __________ How?

Do you think most substances are like water or like paraffin in this regard?

What is the biological significance of the behavior or ice and water? How would life be affected if ice sank and lakes froze over from the bottom up?

How would life be affected if the melting point of water were the same as paraffin?

B. Surface Tension & Adhesion

3. Drop Behavior - Water on Penny.

Drop water into a small (10 ml) graduated cylinder with a medicine dropper, counting each drop. How many drops, of the size produced by your medicine dropper, are in each cc of water? ______

Conversely, how much water is in each drop? _________________

Now, let's see how many drops can you get on the surface of a penny before it overflows. (What do you predict?) ___________

Drop water from the dropper onto a penny, keeping careful count of each drop. Draw a diagram showing the shape of the water on the penny after one drop, when the penny is about half full, and just before it overflows.

one drop

half full
____ drops

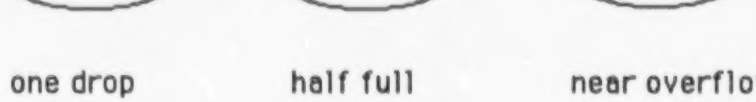
near overflowing
____ drops

How many drops were you able to place on the surface of the penny before it overflowed? ____________

4. Effects of Detergent

Spread one drop of detergent solution on the surface of a dry penny. How many drops do you think this penny will hold? (more, less, or the same as the first penny?) ______________________

Using the same dropper, add drops of water to the penny surface as before. Keep careful count of the number of drops, and draw the water on the penny after one drop, about half full, and just before overflowing.



one drop

half full
____ drops

near overflowing
____ drops

How many drops were you able to place on the penny before it overflowed this time? ____________

Explain what effect the detergent had. ____________________

____

5. Drop Shape on Glass and Wax Paper.

Drop water onto (a) a piece of wax paper and (b) a glass slide. Draw the shape of the drop on each surface.

________ ________

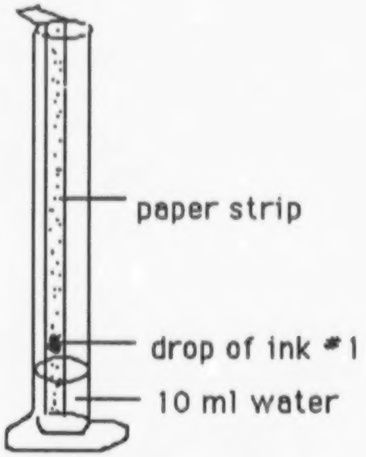
wax paper glass

Explain the differences in drop behavior in terms of the formation (or absence) of hydrogen bonds between molecules.

C. Water, Paper, & Other Molecules.

Water climbs to the tops of tall trees. It will also climb up paper, and often the migrating water will carry other molecules along with it. The distance traveled by these other molecules will vary with their size and charge. How quickly will water climb a strip of paper? What can it carry along with it?

To find out, tear off a strip of chromatography paper that is just long enough to hang over the side of a 50 ml graduated cylinder and and reach to the bottom.



50 ml graduated cylinder

Run the paper strip along the edge of a scissors to take the curl out of it. Then place a single small drop of ink # 1 on the paper, about one inch from the bottom, and let it dry completely. Finally, place 10 ml of water in the graduated cylinder and place the strip of paper in the cylinder so that the bottom end is immersed in water and the drop of ink is just above the surface of the water. Fold the paper over the side. Note the time. __________ Watch at about 5 minute intervals until the water climbs to the top. Note the time, remove from the water, and let dry. How did the ink change?

Time (minutes)	Distance (inches)
5	____________
10	____________
15	____________
20	____________
25	____________
30	____________

Draw the pattern on the paper. How do you explain the results??

D. Cohesion

1. Water & Oil. Place 8 ml. water in a 10 ml. graduated cylinder and gently add 2 ml. of cooking oil by tilting the cylinder of water slightly and letting the oil run slowly down the inside of the cylinder. What will happen to the oil? (Predict! by choosing a, b, c, d, or e below)

a. the oil will float on top of the water
b. the oil will sink to the bottom of the water
c. the oil will dissolve in the water
d. the oil will become mixed up with the water
e. other (what?)

Now do the experiment and record your results. _______________

Save this graduated cylinder with its contents and get a clean 10 ml. cylinder for the next experiment.

2. Oil & Water. Place 8 ml. cooking oil in a 10 ml. graduated cylinder and gently add 2 ml. of water by tilting the cylinder of oil slightly and letting the water run slowly down the inside of the cylinder. What will happen to the water? (Predict! by choosing a, b, c, d, or e below)

a. the water will float on top of the oil
b. the water will sink to the bottom of the oil
c. the water will dissolve in the oil
d. the water will become mixed up with the oil
e. other (what?)

Now do the experiment and record your results. _______________

What is lighter (less dense), oil or water? __________________

This characteristic behavior of water and oil is of critical importance for livings things, determining many properties of the cell. Can you explain how?

3. Water, Oil, and Dye. If you add a few drops of a water-based dye solution to each of the graduate cylinders from (1) and (2) above, what will happen? Predict: ______________________

Perform the experiment to see if you were correct. How does the dye behave in each cylinder? Does it diffuse into the oil? Into the water?

Stir the contents of each cylinder with a stirring rod and then let it sit. Will the contents remain mixed? Observe what happens and explain your results.

4. Sheen. Take a clean beaker of water. What will happen if you add one small drop of oil to the water using a medicine dropper? (Predict!)

Do this experiment. Can you see the oil? Was your prediction correct? Add more drops of oil if necessary to see it clearly. Describe.

Add a drop of detergent to the beaker. What will the detergent do? Record your results and explain how the detergent works in molecular terms.

Explain some of the consequences of oil spills in the sea. What effects do they have on sea life and bird life, and what methods are used to 'clean up' oil spills?

Organizing Your Knowledge Again

Various terms are defined below that are useful in understanding and describing water. For each term, list one or more experiments you may have performed that illustrate the phenomenon described (there may not be an experiment to correspond to every term, and an experiment may apply more than once). Also, wherever possible, describe at least one observation you have made outside the laboratory that illustrates each phenomenon.

1. polarity - a molecule has polarity if it has an unequal distribution of charge; water is polar because the oxygen has a partial negative charge and the hydrogen atoms each have a partial positive charge; polar molecules interact with other polar and charged molecules

2. hydrogen bonds - weak bonds that form within or between molecules, specifically involving an atom that has a partial negative charge and another atom (in the same or a different molecule) having a partial positive charge; results in 'sticking together' of molecules, called cohesion if the molecules are the same and adhesion if the molecules are different from one another

3. boiling point - the temperature at which a liquid turns into a gas; for water, the temperature is relatively high, 100 ^{o} C. or 212 ^{o} F.

4. cohesion - the tendency of molecules to be attracted to one another, as occurs with polar water molecules; such molecules form a highly dynamic structure involving many rapidly breaking and forming hydrogen bonds; water has high cohesiveness or cohesion, enough so that insects can walk on water

5. surface tension - the tendency of molecules of a substance to stick together at the surface, as occurs with water due to its polarity and hydrogen bonding; a special case of cohesion

6. adhesion - the sticking together of molecules of different substances, as occurs with polar molecules, such as water adhering to paper

7. capillary action - the tendency of one substance to move along the surface of another, even in spite of gravitational or other forces, due to adhesion

8. specific heat - the amount of heat energy required to disrupt the organization of a substance; water has a high specific heat due to the high number of hydrogen bonds among its molecules (the specific heat of water is 2X that of most carbon compounds and 9X that of iron); alcohol has a lower specific heat and thus evaporates more quickly

9. thermal conductivity - ability to conduct heat away from another object; air is a poor thermal conductor, water is an efficient thermal conductor because of its high specific heat

10. freezing point - the temperature at which a substance turns from a liquid into a solid (and vice versa); water is different from nearly all other substances because it is densest in liquid form at about 4 ^{o} C., less dense in solid form

11. amphipathic - a molecule that is "water-loving" (i.e.,, polar) at one end and "water-hating" (i.e.,, nonpolar) at the other end; helps hold polar and nonpolar molecules together; characteristic of detergents and of the phospholipid molecules in the membranes of living things

Cells

Equipment
- microscope
- alcohol burner
- beaker
- hot plate
- knife
- cutting board
- scissors
- lens tissue

Supplies
- toothpicks
- methylene blue stain
- cover lips
- onion
- droppers
- salt
- immersion oil
- saline
- toothpicks

A. Organizing Initial Knowledge

I. It is estimated that there are 300,000 different kinds of organisms (species) in the world today (and that about 100 times that number, or 3,000,000 different kinds of organisms, have lived on the surface of the earth to date) . These organisms can be divided into five kingdoms.

A. **Animalia** (animals)
B. **Plantae** (plants)
C. **Fungi** (fungus and molds)
D. **Protista** (mostly unicellular animal-like and plant-like organisms)
E. **Monera** - the unicellular bacteria

2. Every living organism is made up of cells. According to the cell theory,
   a. **All living things are composed of cells.**
   b. **All cells come from pre-existing cells.**
   c. **Cells are the smallest divisible units of life.**

3. Cells of all the higher organisms (A-D above) have certain features in common including
   - cell membrane,
   - nucleus containing hereditary material (DNA) organized in discrete bodies called chromosomes
   - cytoplasm containing a cytoskeleton and a variety of organelles
   - mitochondria (singular, mitochondrion), where respiration takes place (that is, the 'burning' or oxidation of sugar, a process that uses oxygen and produces carbon dioxide)

4. In addition, there are certain cell features that are unique to and characteristic of organisms in each different kingdom. Plant cells, for instance, have a cell wall, chloroplasts, and a large central vacuole, whereas animal cells do not have these features.

B. Observations

1 Examining a Microscope

Obtain a microscope from the cabinet. Always carry a microscope with two hands, one on the back and one underneath the scope.

Examine the scope. Find the eyepiece (the lens at the top). Does your eyepiece magnify 10X? Look at the objective lenses at the bottom of the tube, on the rotating plate. What magnifications do your lenses have?

a)
b)
c)

An eyepiece with a magnification of 10X and an objective lens with a magnification of 40X produce 400X total magnification (10 x 40). What total magnifications are available on your microscope?

a)
b)
c)

2. Using a Microscope.

1. Set up a microscope lamp so that it shines on the mirror at the bottom of the scope (this does not apply if your scope has a built-in light).

2. Adjust the mirror to get the maximum concentration of light in the lens just below the stage (the stage is the place where you place a slide), the lens that you can see when looking through the circular opening in the stage.

3. You may have to open the diaphragm below the stage to get maximum light through (the diaphragm is adjusted with a small handle that protrudes from the central column below the stage; open and close it).

4. Place a slide on the stage of the scope and hold it in place with the arms that are attached to the microscope stage.

5. Looking from the side, turn the knob of the lens to move the shaft of the microscope downward until the lens just touches the slide.

6. Then look through the top eye piece and very slowly turn the knob in the opposite direction, raising the lens, until the slide comes into focus.

3. Examining Prepared Slides

Examine the prepared slides. Draw a picture, as detailed as possible, of each type of cell. Beside each picture, note its magnification. Also note any distinguishing features. You will get the best (largest) image with 100X magnification. To use this, you MUST use immersion oil. When you are done, carefully clean the oil off each slide and off the objective lens.

a. Human Tissues

(1) Human blood magnification: ________ features:	(2) Squamous epithelium (e.g., skin cells) magnification: ___________ features:
(3) Ciliated epithelium (e.g., digestive tract, nose) magnification: ________ features:	(4) Human muscle magnification: ___________ features:
(5) Human mammary gland	(6) Human Sperm

magnification: _________ features:	magnification: ____________ features:

b. Plant Tissues

(7) Mixed pollen magnification: _________ features:	(8) Lillium anther & microspores magnification: ____________ features:
(9) Lillium pollen tubes magnification: _________ features:	(6) Other (what?) magnification: ____________ features:

4. Preparing & Examining Your Own Slides:

You can have students prepare your own slides as follows:

4a. Cheek cells (Animalia)

Take a clean toothpick and scrape some cells from the inside of your cheek. Spread the cells in a small drop of saline solution on a slide. Add a very small drop of methylene blue stain and stir the cell/saline/stain mixture with a toothpick (the drop should be light blue). Clean a cover slip with lens paper and hold it at an angle, with one edge resting against the edge of the water drop on the slide. Let the cover slip fall gently onto the drop. Examine under the microscope. Can you see the cell membrane, nucleus, and nuclear membrane? Any other details? Draw cheek cells and label the parts you can identify.

Human Cheek Cells magnification: ________ features

5b. Onion cells (Plantae)

Slice an onion about a quarter inch thick. Carefully remove the thin membrane (sheet of cells) that occurs between two rings of the onion. Spread a piece of the membrane on a shallow drop of saline on a slide. Fix the cells by passing the bottom of the slide over an alcohol flame. Add a drop of stain. Examine under the microscope. How does an onion cell differ from a a cheek cell? Is it a different shape? Can you see a cell wall? Draw and label onion cells.

Onion Cells magnification: ________ features:

Recommended: Read 58-69 (cells)

15. Animal Behavior

Animals are continually interacting with their environment. The environment includes both living and non-living elements with which the organism has some form of contact.

The way in which each organism responds to its environment is a matter of considerable interest. Studying an organism's behavior helps us to understand and predict how that organism will react in future situations.

In recording animal behavior, you are to describe behaviors rather than record inferences. For example, supposing you are watching a pitcher on the mound during a ball game. If you report that she is getting ready to pitch the ball, you are making an inference. If you describe the way she swings her right arm around, lifts her left foot off the ground, and otherwise postures, you are describing her behaviors.

Likewise, if you are studying your dog and you report that the dog needs to go out, you are reporting an inference. What we are interested in are the detailed behaviors which lead you to that conclusion. You might, for example, report that the dog is whimpering, that it walks back and forth between you and the door, that it scratches the door with its right paw, and so on.

In recording behaviors, you should avoid making value judgements as well as drawing conclusions. You may not be inclined to poke your butt in the air toward people who are watching you, but that doesn't mean that is an inappropriate behavior for a monkey. Try to be an accepting, objective, and thorough reporter of the behaviors you see.

To do this exercise, follow the steps below.

* * *

Step 1. Domestic dogs, and cats, humans, subhuman primates, and giraffes are excluded as objects of this experiment, as are any pets you have at home. A confined animal might be easier to work with than one that might fly away or slip off into the bushes, but it should not be in such a small enclosure that interesting behaviors are prevented. You might find an interesting subject at Sea World, Scripps Aquarium, the Zoo, a city street (pigeons), or on a ranch. You will have free access to the zoo.

Step 2. Each team member should select a different time of day to observe the animal:

early morning (9-11 a.m.),
late morning (11 a.m.-1 p.m.),
early afternoon (1-3 p.m.), or
late afternoon (3-5 p.m.).

Step 3. Describe the animal in general. Identify the animal in your report and give a complete description of it. Consider how big it is, its approximate weight, the relative sizes of its various body parts, its outer covering (fur, skin, feathers, scales, etc.), the shape of its head, mouth, eyes, nose, and teeth or beak, its appendages (number of legs, arms, fingers, toes, antennae, etc.), its coloration, and so on.

Step 4. Identify specific markings or other characteristics that make your animal uniquely identifiable, that set it apart from other animals of its type.

Step 5. Record the animal's behavior for two continuous hours. First find a comfortable vantage point for studying the animal. Record the animal's behavior giving as *COMPLETE and succinct a description of what the animal is doing as possible.* Behaviors may include such things as walking, running, jumping, standing, sitting, lying down, pacing, nose twitching, tail flicking, gazing, blinking, chewing, sniffing, licking, and many others.

Use a stopwatch or wristwatch to record the time of each behavior at intervals of 1-5 minutes. Take care not to draw inferences or form judgements. Simply **describe**, in a thorough, objective, and simple manner. Record your observations on lined paper. Note the time throughout the observation period at 1-5 minute intervals.

Step 6. Summarize the amount of time spent in each behavior in a pie graph. Include pie graphs created by other members of your group and briefly discuss changes in behavior at different times of day.

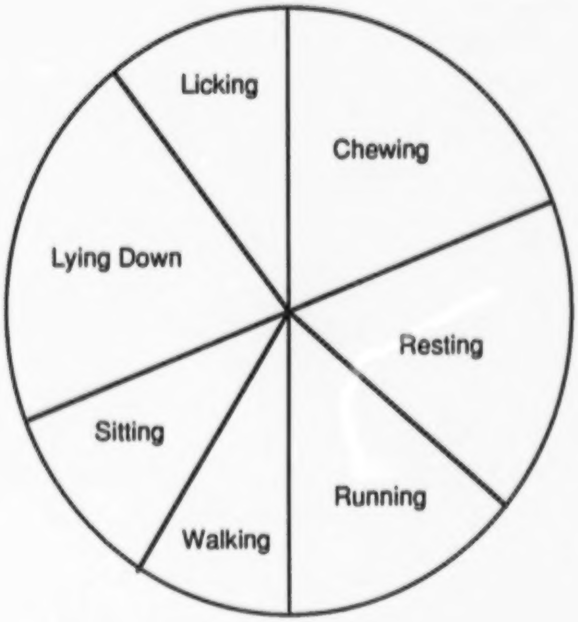


Table 4.1: Behavior Pattern During Two Hour Interval
1-3 p.m.

Step. 7. Summarize your observations in a 2-3 page report according to the following outline.

Individual Animal Behavior Report

1. Your name
2. Date animal was observed
3. Time of Observation: Start __________ Finish ______________
4. Animal Type
5. General description of animal (include picture or sketch)
6. Specific identifying features of animal.
7. Description of enclosure, including size, shape, and structure.(sketch)
8. Description of other animals in enclosure.
9. Record of behaviors (raw data)
10. Summary of behavior (description and inference)
11. Graphic Summaries and Discussion of Changes in Activities during Day

8. Present a 3 minute summary of your observations in class on 4/11/91.

Mitosis
(Chromosome Replication & Division)

Supplies:

plastic knives, forks and spoons in red and white
butcher paper
scissors
rubber bands

An identical copy of your hereditary material, your blueprint, is found in the nucleus of every cell in your body. The blueprint is organized into 46 'chapters' or parts known as chromosomes. Each chromosome may contain a thousand genes or so. Every time a cell divides, every chromosome must be carefully replicated (copied) and then carefully distributed so that each daughter cell gets an identical copy.

There are two different mechanisms of cell division in your body. Most of your cells (and those of all other organisms except the bacteria) divide by mitosis. The other mechanism is meiosis, which occurs in specialized cells that are found in gonads (sex organs) and that produce gametes (reproductive cells, the sperm, eggs, and pollen).

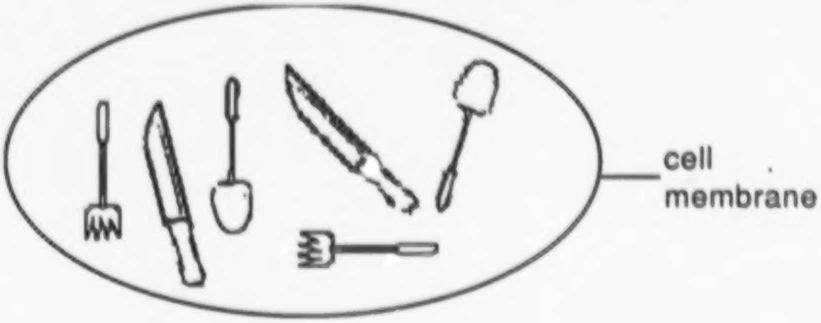
There are a few facts to know about MITOSIS:

a. there is one complete replication of the chromosomes
b. there is one cell division
c. chromosomes do not pair
d. two daughter cells are produced
e. daughter cells are identical to the parent cell

In this lab you will explore how chromosomes are copied and distributed to each daughter cell in a precise way. You will observe mitosis in the *Triffle*, a mythical creature with six chromosomes. You will work out each step with your knife, fork and spoon chromosomes, and draw the results on your lab. Cut out a large circle of butcher paper for your nucleus (big enough to hold 12 eating utensils) and an even larger circle for your cell.

1. Begin with a nucleus containing six chromosomes represented by two forks (one red & one white), two knives (one red & one white), and two spoons (one red & one white). This is the 'diploid chromosome set' characteristic of a *Triffle*. There are two of each kind of chromosome. The two similar chromosomes are NOT paired. The three similar chromosome sets have the same size and shape but they are not identical (in this case,

they are different colors; in the body, they may contain different versions of many of the genes that are contained in each chromosome).



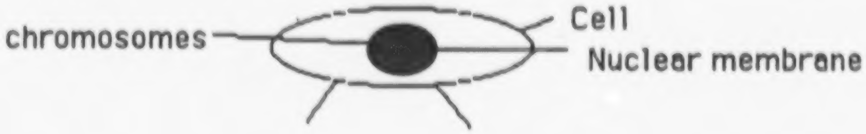
How many chromosomes are in a diploid human cell? _______

2. Each chromosome makes an exact copy of itself by DNA synthesis. The two identical copies, the 'parent chromosome' and the 'daughter chromosome', remain attached at point called the centromere. You can 'synthesize DNA' in your nucleus by taking six chromosomes from the available chromosome pool to match the set you already have and attaching each one to a matching chromosome in your nucleus with an elastic band (which will represent the centromere). In this way, replicate each of your chromosomes.
How many chromosome copies do you have in your nucleus now? ________

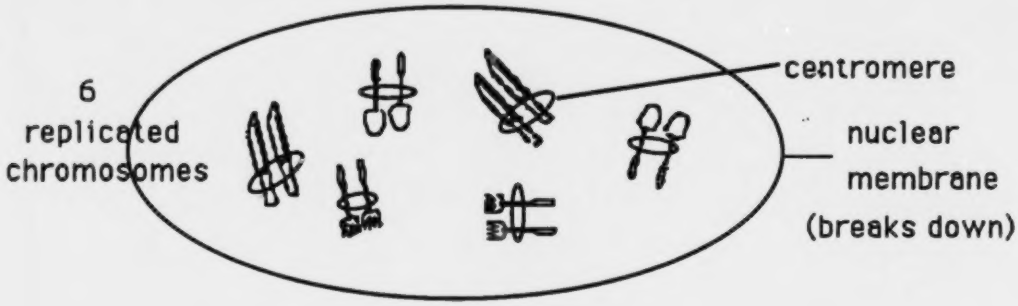
How many chromosome copies are in a human cell after replication? ______

3. Now the cell will go through the stages of mitosis.

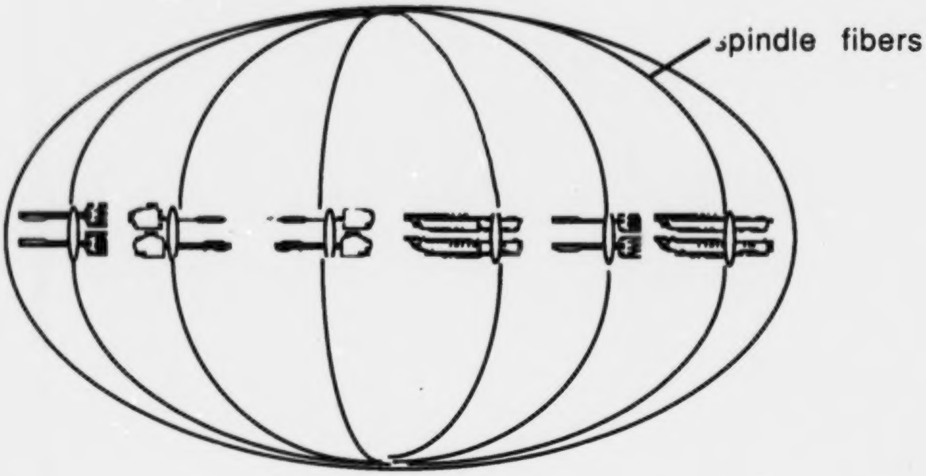
Interphase - chromosomes extended and not visible in the light microscope; chromosomes replicate



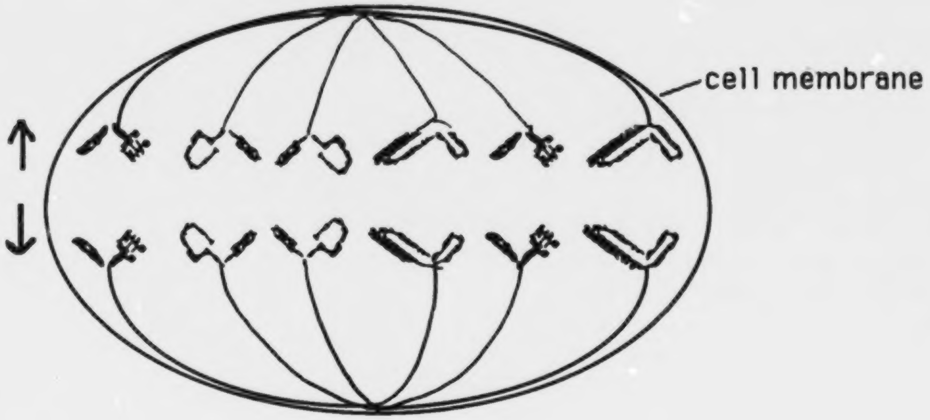
Prophase - replicated chromosomes condense and become visible (chromosomes have replicated already)


6
replicated
chromosomes
centromere
nuclear
membrane
(breaks down)

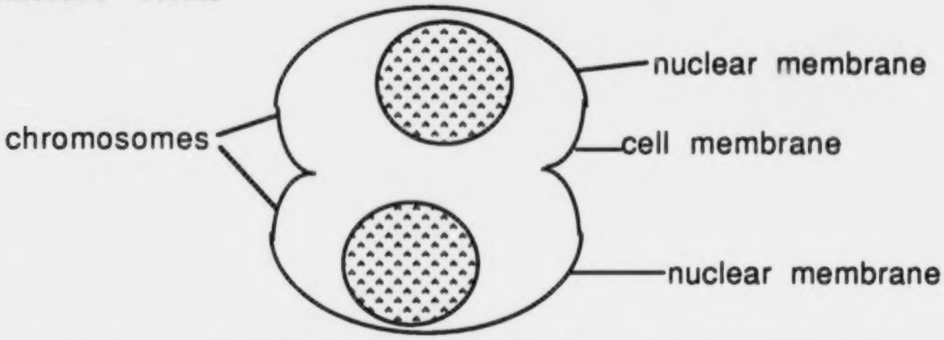
Metaphase - replicated chromosomes line up on metaphase plate; replicated chromosomes are independent of one another; similar chromosomes DO NOT PAIR


spindle fibers

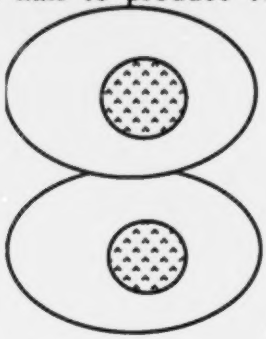
Anaphase - the parent chromosomes and their replica daughter chromosomes separate and move toward opposite poles with help from spindle fibers


cell membrane

Telophase - chromosomes become extended (relaxed) and new nuclear membrane forms


nuclear membrane
chromosomes
cell membrane
nuclear membrane

Cytokinesis - cell pinches in half to produce two daughter cells



Meiosis

A. Goal.
To examine **meiosis,** the specialized cell division that produces the sex cells or gametes (sperm and egg in humans and other animals, pollen and egg in plants). Gametes have half as many chromosomes as other cells in the body.

B. Review:
Chromosomes are composed of DNA and contain the genetic blueprint for an organism. All the organisms in a particular species typically have the same number of chromosomes. All normal human beings, male and female, have exactly the same number of chromosomes (46) in every somatic cell in their body. Each human sperm and egg cell contains one-half the somatic number or 23 chromosomes. When fertilization occurs a sperm and egg unite, producing (restoring) the bodily or somatic chromosome number of 46.

Each species has its own unique chromosome set. The domestic dog has 78 chromosomes, the domestic cat has 38 chromosomes, and the mouse that it chases has 40 chromosomes. In some animals the sexes differ by one chromosome. In the kangaroo rat (Potorous tridactylus apicalis), for example, the male has 13 chromosomes while the female has 12, and in the big fruit-eating bat (Artibeus lituratus), the male has 31 chromosomes and the female 30.

Meiosis is one type of nuclear division that occurs in higher organisms (the other type is mitosis). Meiosis occurs in specialized diploid cells in the reproductive structures of plants and animals.

Meiosis occurs only in cells that have a diploid (2N) chromosome number. Meiosis produces four daughter cells, each with a haploid chromosome number. Each daughter cell is different from the parent cell by virtue of having (a) half its chromosome number and (b) chromosomes that are no longer like those in the parent cell (the genes are reshuffled during meiosis by a process called recombination).

[Mitosis, in contrast, produces two daughter cells from one parent cell; the daughter cells are identical to the parent cell and to each other in terms of both chromosome number and chromosome type.] Mitosis can occur in haploid as well as in diploid cells.

The process of meiosis is similar to mitosis in that:
1) both involve one round of DNA replication (chromosome doubling).

The process of meiosis is different from mitosis in that:
1) similar (homologous) chromosomes pair in meiosis (but not mitosis)
2) homologous chromosomes exchange parts in meiosis (but not mitosis)
3) meiosis involves two cell divisions (whereas mitosis involves one)

C. Self-Test: (check all **correct** responses)

1. What is the specialized cell division called meiosis?
   a. it is the process by which gametes are formed
   b.it is the process by which gametes divide
   c. it is involved in egg and pollen production
   d. it occurs in the anther of a flower
   e. it is how sperm multiply

2. The primary purpose(s) of meiosis is/are to
   a. separate larger from smaller chromosomes
   b. reduce the chromosome number
   c. allow union of cells from different parents without an increase in chromosome number
   d. sort the chromosomes by type
   e. produce genetic variation through reshuffling of genes

3. Meiosis occurs in
   a. all organisms
   b. all diploid organisms
   c. plants and animals
   d. the reproductive structures of higher organisms in which gametes are produced
   e. haploid organisms
   f. the gametes
   g. the ovaries

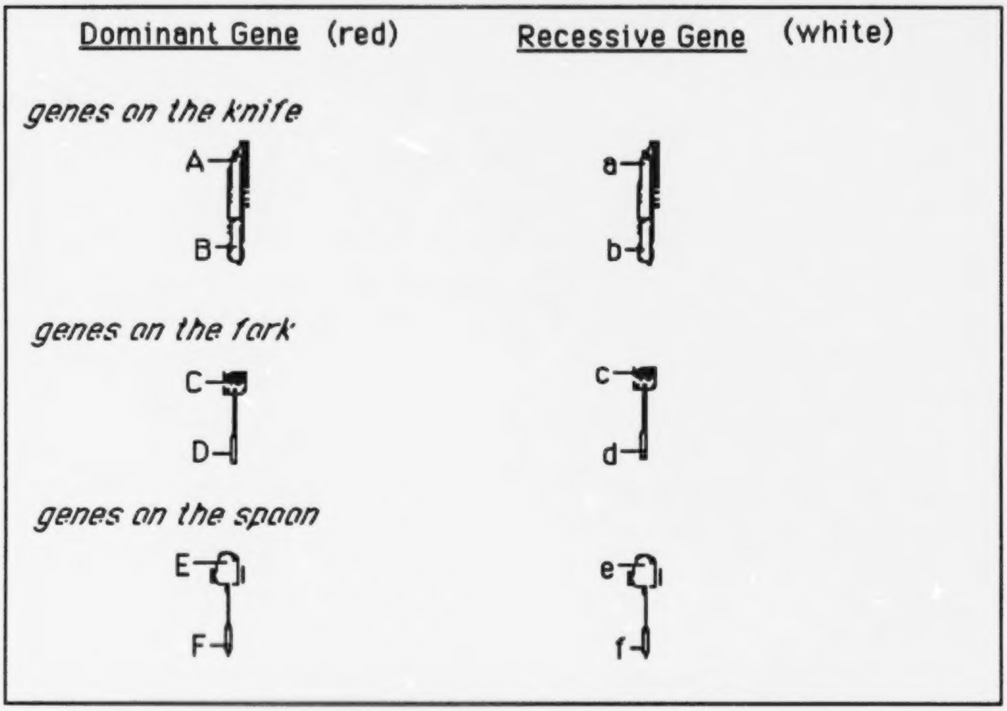
D. Meiosis Exercise

You are going to work through meiosis step by step. Your will be working with the chromosomes of a *'Triffle'*, a mythical organism that has a diploid chromosome number of six. You and a teammate are to complete the following steps.

1. Let three haploid chromosomes be represented by a knife, a fork, and a spoon. Create a diploid nucleus containing two similar but not identical chromosomes of each type - that is
   two knives (one red and the other white),
   two forks (one red and the other white), and
   two spoons (one red and the other white).

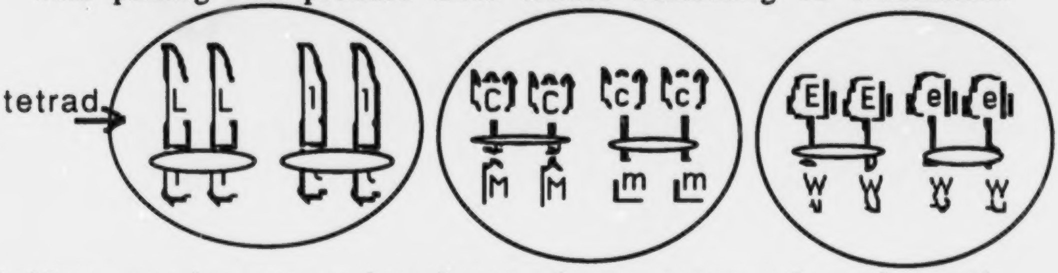
The chromosomes in each pair (that is, the two knives, two forks, and two spoons) are said to be homologous, meaning similar but not necessarily identical. .

2. Label one gene at each end of each chromosome, using masking tape. We'll assume that the traits we studied in humans (and their corresponding genes) are also found in the Triffle.

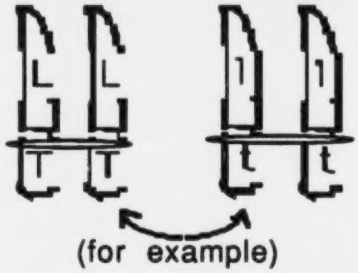


3. Each chromosome then makes an exact copy of itself through DNA replication. Do this by adding a matching chromosome for each of the six chromosomes in your nucleus, labeling the new 'daughter' chromosome with tape so it matches the 'parent' chromosome exactly. Connect daughter and parent chromosomes together with rubber bands. This gives you six replicated chromosomes each containing 2 'sister' chromatids. Trace the six chromosomes on a plain piece of paper and note the genes on each one.

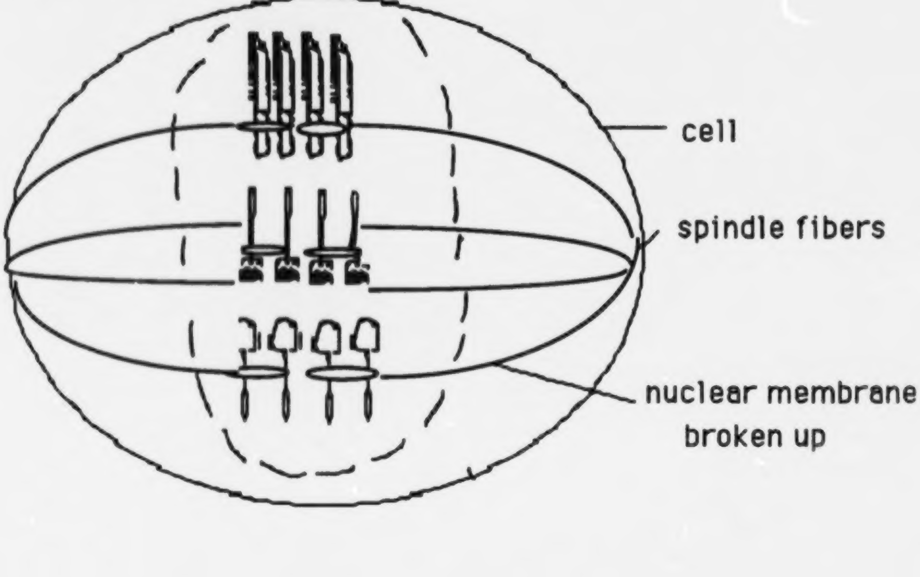
4. Each replicated chromosome then pairs with its homologous replicated chromosome. That is, the replicated knife pairs with the replicated knife, the replicated fork pairs with the replicated fork, and so on. This pairing will produce three tetrads containing 12 chromatids.


tetrad
L
L
l
l
C
C
c
c
M
M
m
m
E
E
e
e
W
W
W
W

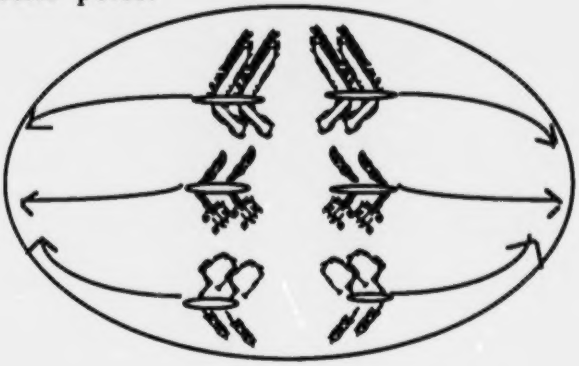
5. Next, crossing over and exchange of parts occurs. In each tetrad, switch one pair of genes between two non-sister (or non-identical) chromatids, by swapping pieces of tape. That is, swap a gene from a red knife with a gene in the corresponding position on a white knife.


L
L
l
l
T
T
t
t
(for example)

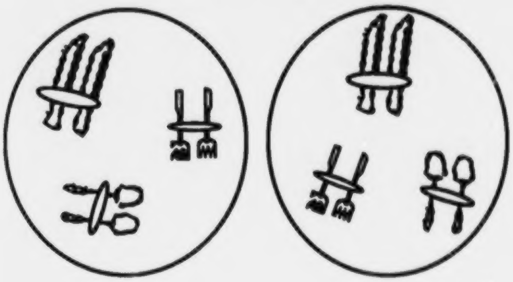
6. Line the tetrads end to end across the center of the cell. (The nuclear membrane has broken up.)


cell
spindle fibers
nuclear membrane
broken up

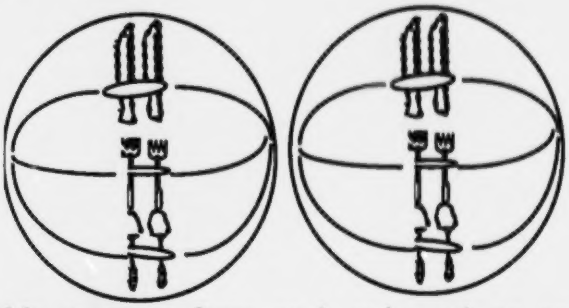
7. Each tetrad divides in half, with homologous replicated chromosomes going to opposite poles.



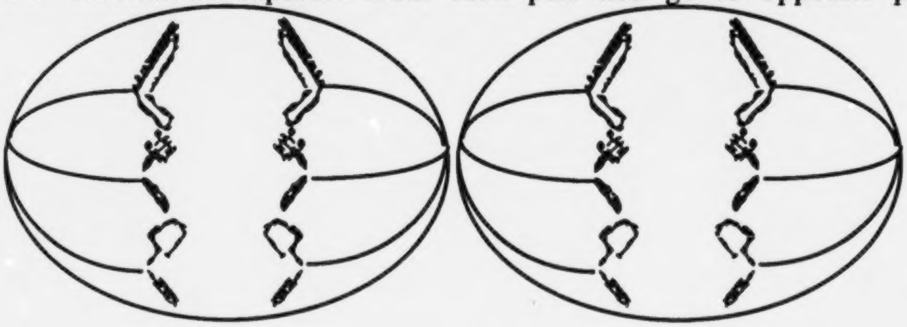
8. The cell divides to form two daughter cells.



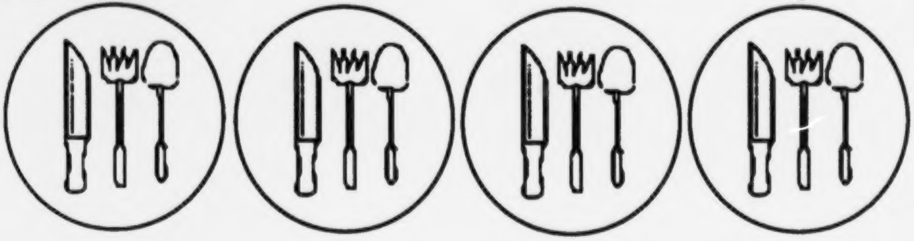
9. The replicated chromosomes (half-tetrads) line up end to end in the center of the two cells.



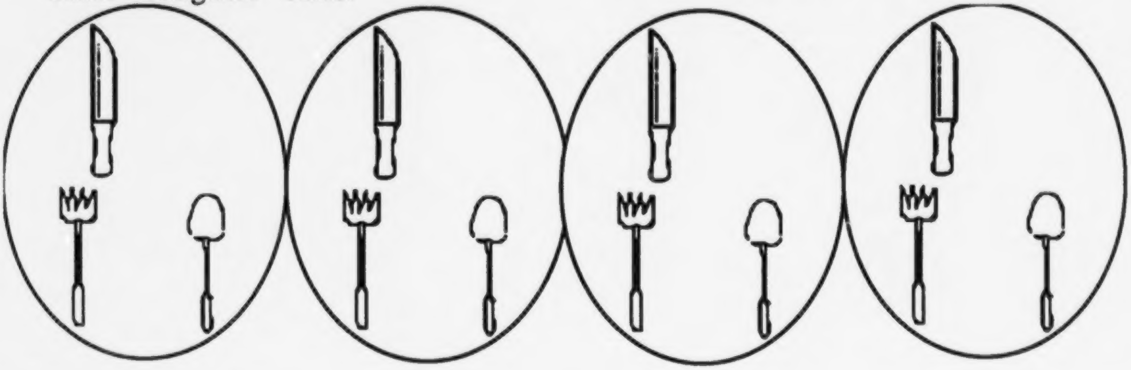
10. The chromatids separate from each pair and go to opposite poles.



11. The two cells divide again, producing two cells each or a total of four daughter cells.



12. Each daughter cell contains a haploid set of chromosomes (that is, a knife, fork, and spoon). Each daughter cell contains one or more chromosomes that is different from those in the parent cell and in other daughter cells.



Label the genes on each chromosome.
Write the genotype for each cell below.

__________ __________ __________ __________

Basic Processes
Evaporation, Diffusion, Solution, Filtration, Osmosis

Equipment

- Hot plate
- beakers
- stirring rods
- Short-stemmed filters
- Tongs
- Clock or watch
- Refrigerator

Supplies

- After-shave lotion
- Ground coffee
- Salt
- Sugar
- Coffee filters
- Water
- Carrots
- Celery
- Raisins
- Egg

Organizing Initial Knowledge

1. The simplest kinds of matter are the elements shown in the periodic table (attached). These occur naturally on earth (except for a few new ones, which were created artificially). The elements (and all other substances) are made up of particles. The particles that make up a given element are all identical to one another. Each of the 103 elements is made of a different kind of particle. The particles that compose elements are called atoms.

2. Scientists studied elements and their properties for many many years. The 'atom' is part of a 'theory of substances' that has been developed; the 'idea' of an atom helps us explain and understand many of the properties of elements.

3. Atoms are basic building blocks that can combine with one another in an enormous variety of patterns. They link together in regular ways to form other particles called molecules. A molecule by definition contains more than one kind of atom. Millions of the same kind of molecules together form a homogeneous substance such as water or absolute alcohol. (A homogeneous substance is one that is the same throughout).

4. Molecules are much smaller than living cells, and we know that a living cell is usually so small it can only be seen with a microscope. Millions of atoms and molecules can be found inside a single living cell. How can scientists study anything that is this tiny? The answer is, in many different ways, such as by

looking at many molecules together and
looking indirectly at the effects and behaviors of molecules.

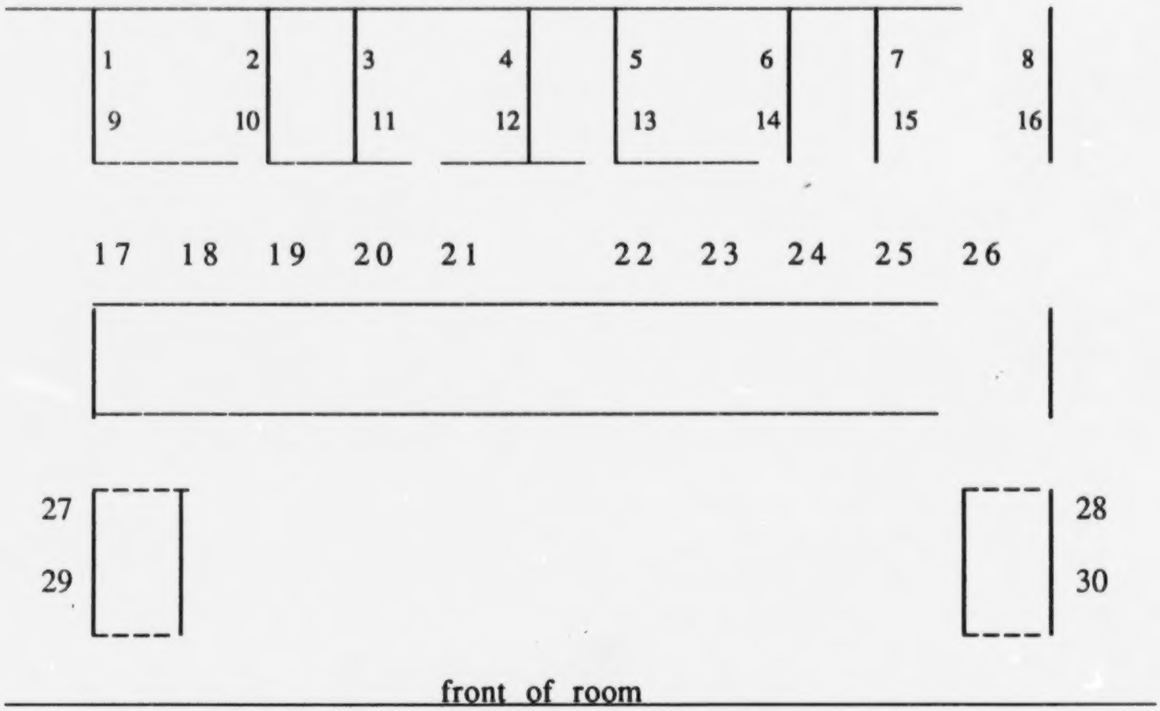
5. In this lab, we will examine the particulate nature of substances in some very basic ways.

Experiments

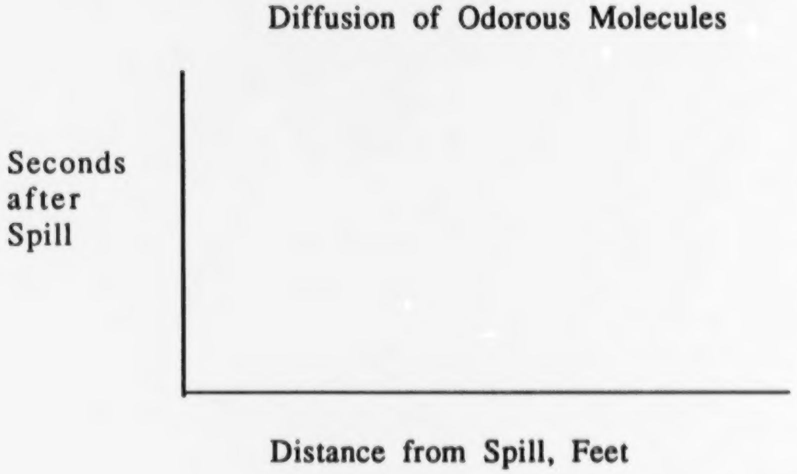
1. Diffusion of Gases. Watching the clock or a stopwatch, spill after-shave lotion or another harmless, pungent liquid in one corner of the room. Note the time it takes for the odor to reach each student in the room. An approximate floor plan and distribution of students (indicated by numbers) is shown below. Record the time that the lotion is spilled and the time at which it reaches each person or group.

NS 412C Laboratory windows/outside

Time Lotion is spilled: ________



Make a graph in which you plot the time it took for the odor to reach a particular area.(in seconds) against the distance of that area from the spill 'as the crow flies.' Represent each intersection of time and distance with a dot.



Sometimes several areas that are the same distance away receive the odor at different times. How would you account for that?

2. **Diffusion through a Membrane.** Boil some water in a large (250-500 ml) beaker on a hot plate. Place a short-stemmed glass or plastic funnel in each of two smaller beaker, so the sides of the funnels (not the bottoms) rest on the beakers. Place a coffee filter in each funnel and add a spoonful of ground coffee to each. Pour boiling water through the grounds into one beaker. Pour cold water over the grounds in the other beaker.

Prediction (What do you anticipate will happen?)

Observation (What do you actually observe?)

Does the water pass through the membranes?

What color is it in each beaker?

Do the coffee grounds dissolve and pass through the membrane?

Interpretation: Write the best possible explanation you can for your observations.

Extrapolation. Can you describe several places in the human body where water passes across a membrane? What is the temperature of water in the human body?

3. **Solution - Sugar.** Place 25 ml cold water in each of two 50 ml beakers. Place one beaker on a hot plate and bring it to a boil. Add sugar to the cold beaker, one small level spoonful at a time. Stir until dissolved. Then add another level spoonful and stir. Continue until you reach the saturation point (when no more sugar will dissolve). How many spoonfuls of sugar dissolved in 25 ml cold water?

Similarly, add one level spoonful of sugar at a time to the boiling water, stirring after each addition. Does the water in the two beakers dissolve the same amount of sugar? Explain your results.

What happens when a substance 'dissolves'?

Name two important differences that you have observed, comparing the sugar and the coffee grounds.

How does temperature affect the solution process?

Extrapolation. Name places in the human body where sugar is dissolved in water. Where sugar diffuses across a membrane.

4. Evaporation - Salt. Fill a small beaker about half full of water and add salt, stirring until dissolved. Continue adding salt until you reach the saturation point. Place the beaker containing salt water on the hot plate and boil until nearly dry. Watch closely so you can remove the beaker from the heat as soon as the solution is almost (but not quite) dried (Be careful - the beaker will crack if you do not remove it immediately!)

Where does the water go during evaporation?

Does the salt also evaporate? Can you explain what happens?

How important is the heat energy supplied by the hot plate for the evaporation process? Would evaporation occur without a hot plate? How long would it take?

Explain as precisely as you can how the heat energy influences the reaction.

Extrapolation. Name places in the human body where evaporation occurs. What purpose does it serve?

5. **Osmosis.** Place a stalk of celery (about 3"), a carrot (about 3"), and/or a dozen raisins in each of two beakers. To one beaker add cold fresh water to cover the vegetables. To the other beaker, add cold salt water (saturated) to cover the vegetables. Put your name on each beaker and place the beakers in the refrigerator for observation next time. What will happen to the vegetables in each beaker? To the raisins? Why?

Prediction

fresh water -

salt water -

Observation
fresh water -

salt water -

Interpretation. What produced the changes you observed? Was diffusion involved? Diffusion across a membrane? If so, what substance moved, and in what direction?
fresh water -

salt water -

Extrapolation. Can humans survive by drinking salt water such as ocean water? Why or why not?

6. Dissolving Eggshell. Submerge an egg in vinegar and observe it next time you come to class. What changes do you expect to see?

Organizing New Knowledge - Can you answer these questions in another way now?

1. How do we study molecules we cannot see?

2. What are the phases of matter? Describe each phase for water. Do all phases of water exist in the human body?

3. How do molecules behave (move) in each phase? What is required for changing phases?

4. What does it mean to dissolve? Do all molecules dissolve in water?

5. Can molecules pass through membranes? What kinds of molecules? What kinds of membranes?

6. Can you define each term in the title of this lab?

Prepare a graph to summarize the results of each experiment you have performed in this lab.

Recommended: Read pages 26-41 and relate the reading to your laboratory.

18. Animal Breathing & Respiration

What do you already know about breathing?

Newborn babies emerge from the womb and take their first breathe, sometimes prompted by a slap on the bottom. Breathing continues from then on, at a moderately steady rate. Any interruption in our breathing (for a few minutes or more) can be very bad for our health! Why do we breathe so much? What does breathing do for us?

What do we breathe? What goes in? What comes out?

How do we breathe? What body parts and systems are involved? What are the mechanisms?

If breathing is interrupted for too long, what is usually the first part of the body to be damaged?

Do all animals breathe? Do plants breathe, too? What about amoebas?

Are respiration and breathing the same thing? If not, how do they differ?

Breathing

Equipment & Supplies

- Stopwatch
- ruler
- grease marking pencil
- disposable mouthpieces (straws)
- graduated cylinder
- steel wool
- plastic container for steel wool experiment
- water
- large jug
- large plastic dish pan
- rubber tubing (about 1 meter)
- limewater
- lab air supply

1. Not Breathing - Absence of Air (requires watch with second hand)

How long can you hold your breath? (prediction):

Work with a partner. One of you is to hold your breath as long as you can while the other is to time the non-breathing interval with a stopwatch. Imagine that you are far under water and you must hold your breath until you reach the surface. Repeat three times for each person, record all three measures, and average them (add them up and divide by three).

Time Breath Held, minutes & seconds

	Subject: ______	Subject: ______
Trial 1	________________	________________
Trial 2	________________	________________
Trial 3	________________	________________
Total	________________	________________
Average	________________	________________

Create a table on the board, summarizing the average time that each student were able to hold their breath (in minutes and seconds). Compare the times for class members. Are there differences among the various individuals? Name three factors that might cause such differences.

2. Breathing - How much air do you take in to your body?

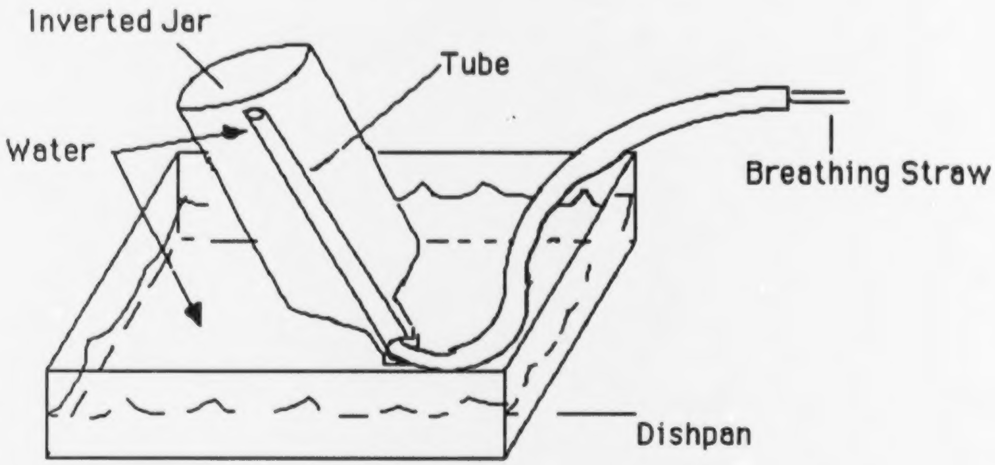
Life on earth is immersed in a sea of gases called 'air'. On good days the air is invisible - we can't see it and we can't smell it, and we are so used to the feel of it that it may be difficult to describe what it feels like. When we breathe, some of this air enters our bodies. Can you estimate how much air you take in when you take the deepest breathe you can?

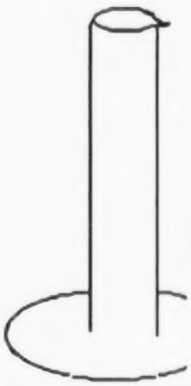
Estimate: ________ cubic in or ________ cubic cm

How can we measure the volume of air? One way to do it is to see how much water is displaced by the air.that we expel from our lungs. A particular volume of air should displace an equivalent volume of water.

Work in groups of four for this experiment. For each person, measure the volume of air expelled from the lungs after taking the deepest breath you can and breathing out as completely and deeply as you can.

Method:





1. Fill the dish pan about 1/3 full of water.
2. Fill the large jug with water to the very top.
3. Hold your hand over the mouth of the jug, invert it (turn it upside down), and place it in the pan of water. DO NOT LET THE NECK OF THE JUG RISE ABOVE THE WATER LEVEL IN THE

PAN OR YOU WILL GET WET! Hold the jug carefully - it will be unstable and will tip easily.

4. As one team member holds the jug in the pan at a slight angle another team member inserts one end of the rubber tubing into the container, all the way up to the bottom of the jug (now at the top). Keep the other end of the tubing pinched off.
5. Insert a clean mouthpiece into the free end of the tubing. One team member should then take a very deep breath and slowly but completely exhale into the tubing, forcing water out of the jug into the pan.
6. When the person can exhale no more, remove the tube from the jug, carefully place your hand over the neck of the jug, and turn it upright.
7. Refill the jug carefully, using a graduated cylinder to measure the volume of water required to fill the jug. Record the volume of water added, which is equivalent to the volume of water displaced, and which therefore provides a measure of the exhaled air.
8. Take two measurements on the first subject. If they differ by no more than 10%, take one measurement on successive subjects to save time (two would be preferred of course).

Observation: Lung Capacity

	Initials	Maximum Volume, cc
Subject 1 (a)	_____	__________
Subject 1 (b)	_____	__________
Subject 2	_____	__________
Subject 3	_____	__________
Subject 4	_____	__________

Create a table on the blackboard summarizing the maximum lung volume for each individual (identify individuals with initials). Is there variation in the group data? Copy the group data from the board into a well-organized table. Convert it into a bar graph.

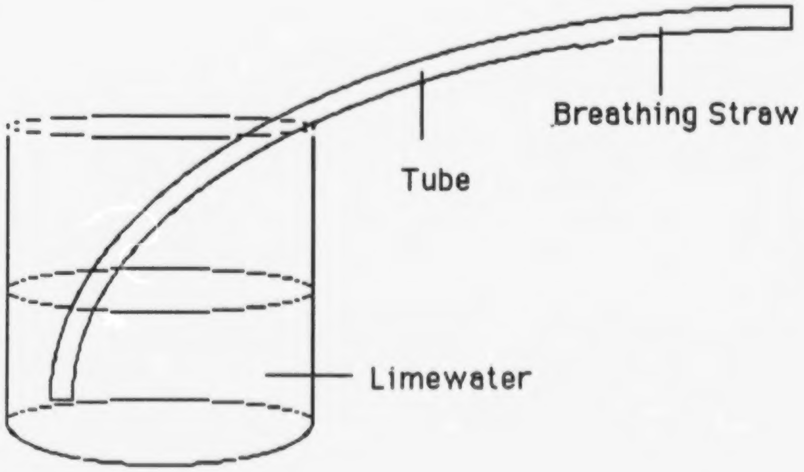
3. How much of the air that you breathe in do you use?

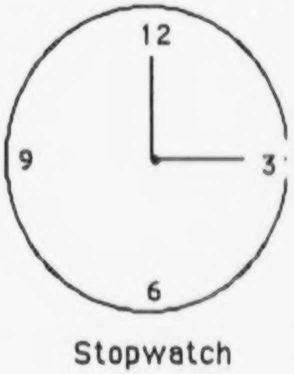
In part 2, we measured the volume of air from the lungs by measuring the amount of water it displaced. In part 3, we will try to determine how much of the air is composed of oxygen, which is the gas that our bodies use. We do this by allowing the oxygen in the air to chemically combine with steel in a container that is closed on all sides, so that no new air can enter. The oxygen combines with steel to form rust. This process is known as 'oxidation'.

Place steel wool in the top of a plastic container filled with air. Turn the container and steel wool upside down in a pan of water. The water will prevent any exchange of gases with the atmosphere. Mark the water level on the container.

As oxygen is removed from the air by combining with the steel wool, water rises in the container to take its place. Thus, this is similar to the displacement test used above. Allow the apparatus to sit until there is no further change in volume (several days or weeks). Cover with plastic wrap to prevent evaporation. Mark the water level at the end of this period. How much less air (approximately) is present at the end than at the beginning of the experiment? This reduction in volume is equivalent to the amount of oxygen removed from the air.

4. How much of the air that you exhaled was produced by your body?





Your body produces a gas - carbon dioxide. Like oxygen, carbon dioxide can be detected chemically. It reacts with limewater to form a visible, insoluble compound, calcium carbonate.

1. Place about 150 ml. of limewater in a small beaker.
2. Place one end of the flexible hose in the bottom of the beaker of limewater and hold it in place.
3. Place a clean mouthpiece on the other end of the tube and gently exhale into the hose until you observe a precipitate in the limewater (inhale if necessary without using the hose). Measure the total time of exhalation (the time spent exhaling into the tube) that is required to form the white precipitate.
4. Obtain another beaker of 150 ml limewater and bubble ordinary air through it for the same length of time you previously exhaled, using the air supply in the lab. Is a white precipitate formed? Is the composition of ordinary air different from the air you exhale?

Questions:

1. When one person gives another artificial respiration, is the air that is administered of higher or lower quality than the air we normally breathe? Compare the two.

2. Distinguish between breathing and respiration.

3. How long can a human being go without oxygen? Can we train ourselves to extend this time? If so, by how long?

4. What specific purpose does the oxygen serve in our bodies? (Note: This is an important concept to understand).

Hint: Air contains about 80 % nitrogen (N_2), 20 % oxygen (O_2), 0.03 % carbon dioxide (CO_2). About 5% of the oxygen is removed on inhalation, and there is about a 5% increase in carbon dioxide in exhaled air.

APPENDIX C.4

Attitude Postsurvey

NS412C

NS412 TEACHING AND LEARNING SURVEY

Put your name or social security number on your answer form. Please feel free to comment on any item on the back of this survey. If you do comment, please put your name or social security number on the questionnaire, too. Thanks.

This survey is for research purposes only. There are NO right or wrong answers and the results will be kept confidential. The survey is designed to learn how you feel about the teaching and learning of Biology. Please read each statement carefully and respond by using the following scale. On your answer sheet, mark in soft lead pencil the letter that most closely represents how you feel about the statement.

****

A) STRONGLY AGREE B) AGREE C) UNDECIDED D) DISAGREE
E) STRONGLY DISAGREE

****

1. Biology research requires making careful observations.
2. A hypothesis is more reliable than a theory because it is based on many observations.
3. The accepted body of knowledge that constitutes biology (the 'facts') keeps changing.
4. Observations made by one skilled research group (or researcher) are just as reliable as an average of observations made by several different skilled research groups.
5. Learning biology is hard work.
6 Biology is best taught in elementary and middle school as a series of activities, observations, and experiments.
7. Biology researchers aim to develop explanatory and predictive models.
8. Reproducibility of observations is not important in biology research.
9. Learning biology means memorizing a lot of unrelated facts.
10. Taking detailed notes is generally not necessary in biology research.
11. One reason for designing experiments is to control the number of conditions that change.
12. Elementary and middle school children should not be expected to learn science process skills such as making observations, classifying, using the metric system, and designing simple experiments.
13. Biology can be incorporated into other subjects taught in elementary and/or middle school, such as reading or arithmetic.
14. Measurement and quantification are not important in biology research.
15. Ideas in science are easier to understand when they form a coherent story.

****

A) STRONGLY AGREE B) AGREE C) UNDECIDED D) DISAGREE
E) STRONGLY DISAGREE

****

16. I could create many different exercises for teaching biology to elementary and/or middle school children using simple, everyday materials.
17. I feel confident about teaching biology at the grade level I plan to teach.
18. I find it difficult to understand newspaper and magazine articles about science.
19. Living things on earth are amazingly interrelated and interdependent.
20. I am not looking forward to teaching biology.
21. I feel confident that I will become a lifelong learner of biology.
22. The grocery store is a great place to obtain supplies for classroom biology experiments.
23. I can improve my own learning through reflection.
24. I would rather have someone else solve a biology problem than to work it out myself.
25. I find biology textbooks difficult to understand.
26. I enjoy reading newspaper and magazine articles about science.
27. Biology is of little relevance to my life.
28. Performing relevant experiments helps me understand and remember biology concepts.
29. I will use biology in many ways throughout my life.
30. Teamwork and collaboration among students can facilitate science learning.

If you want to add any comments, please do so below:

APPENDIX C.5

Final Examination

NS412C

Natural Science 412C Final

I. Multiple Choice. Please mark all correct answers on your test by circling the appropriate letter(s). Note that each question may have more than one correct answer.

1. What molecule, produced by the yeast during respiration, is responsible for making bread rise and for creating bubbles in beer and wine?
   a) carbon dioxide b) lactic acid c) oxygen d) alcohol

2 Oral temperature typically drops with vigorous exercise. The specific mechanism of this temperature drop involves:
- a) increased respiration
- b) increased evaporation
- c) drop in core body temperature
- d) shunting of blood to the intestines

3. Limewater reacts with carbon dioxide to produce calcium carbonate, a white precipitate. When a green plant and a beaker of limewater are sealed in a baggy, a white crust typically forms on the surface of the limewater at night but not during the day because in plants:
- a) respiration occurs at night but not during the day
- b) photosynthesis occurs at night but not during the day
- c) more CO_2 accumulates at night
- d) more CO_2 accumulates during the day

4. Mammals maintain a constant body temperature by . .
- a. altering the flow of blood to the body surface
- b. burning fuel (sugar) to produce heat energy.
- c. insulating their bodies from the environment via fat, fur, etc.
- d. shivering to raise body temperature through kinetic energy.

5. Which of the following flows through the earth's ecosystem, with some being lost at each stage (instead of cycling through living and non-living phases)?
   a. carbon b. nitrogen c. energy d. water

6. Organisms that obtain their energy from dead or decaying matter are called detrivores or decomposers; included in this group are:
   a. vultures b. bacteria c. fungi d. mealworms

7. The primary function served by the anther of a flower is

a. fertilization	b. pollen production
c. germination	d. seed formation

8. Which of the processes below occurs in meiosis (cell division to produce gametes) but NOT in mitosis (somatic cell division)?

- a. two rounds of DNA replication
- b. two cell divisions
- c. exchange of parts between similar (homologous) chromosomes
- d. pairing of similar chromosomes
- e. formation of spindle fibers

9. Starch and cellulose are both built of glucose subunits but they differ in that

- a. starch is easily broken down but cellulose is not
- b. cellulose is easily broken down but starch is not
- c. starch occurs in plants but cellulose occurs in animals
- d. cellulose occurs in plants but starch occurs in animals
- e. starch provides energy while cellulose provides strength

10. Chlorophyll acts in photosynthesis by

- a. binding to oxygen
- b. absorbing light rays
- c. capturing light energy
- d. synthesizing glucose

11. Which processes are important in transpiration (water flow) in plants?

a. evaporation	b. cohesion	c. adhesion
d. capillary action	e. root pressure	

12. Which particles are found in the nucleus of an atom?

a. protons	b. electrons	c. photons
d. neutrons	e. chromosomes	

13. How much larger is a cell nucleus compared to a nucleus of an atom? The cell nucleus is

- a. 10X larger than an atomic nucleus
- b. 100X larger than an atomic nucleus
- c 1000X larger than an atomic nucleus
- d. more than 10,000X larger than an atomic nucleus

14. What advantages does fat have compared to carbohydrates for energy storage? Fat . . .

a. provides more efficient energy storage
b. contains more energy per pound
c. is more soluble in water
d. is more easily converted into sugar

15. Which organs produce digestive enzymes?

a. pancreas b. thymus c. liver d. thyroid e. spleen

16. The kidney is responsible for

a. filtration of the blood
b. shunting liquid waste from the intestines
c. reabsorption of water and needed dissolved substances
d. eliminating the liquid wastes that we ingest

17. Endocrine glands in the human body produce hormones. Hormones are

a. chemical messengers
b. enzymes
c. produced in one part of the body and act in another part
d. important in development

18. The movement of molecules from an area of higher concentration to an area of lower concentration is known as

a. osmosis b. capillary action c. migration d. diffusion

19. The movement of water molecules across a membrane from an area of higher concentration to an area of lower concentration is known as

a. osmosis b. capillary action c. migration d. diffusion

20. Arteries

a. carry blood to the heart
b. are relatively thick-walled
c. run from the heart to a capillary bed
d. usually contain oxygenated blood

21. The advantages for large organisms of being made up of small cells include

a. the ease of gas exchange
b. the large surface to volume ratio
c. the relative ease of diffusion of nutrients and waste molecules
d. the ease of replacement of an individual cell

22. Nitrogen occurs in which class(es) of molecules?
a. Carbohydrates b. Proteins c. Nucleic acids d. Lipids

23. In an atom,
- a. the atomic number is usually twice the atomic weight
- b. the atomic weight is usually twice the atomic number
- c. there are usually equal numbers of protons and neutrons
- d. there are usually equal numbers of protons and electrons

24. There are more species per unit area in which biome?
- a. temperate forest
- b. tundra
- c. chapparal
- d. rain forest
- e. savanna

25. Carbon dioxide is carried in the blood primarily in the
- a. red blood cells
- b. white blood cells
- c. platelets
- d. plasma

26. Which statement(s) is/are true about respiration? Respiration
- a. occurs in plants as well as animals
- b. uses carbon dioxide and produces oxygen
- c. converts energy to a form usable by the cell
- d. is essential for life

II. Short Essay. Please answer briefly, in complete sentences.

27. Describe how the body gets rid of liquid and solid wastes. Your answers should include the names of the organs involved.

a) Liquid wastes (including urea)

b) Solid wastes

28. Select any two of the following three topics: a) heredity, b) respiration, or c) growth & development. Write a concise paragraph about each of the topics you choose. Assume that you are introducing the topic to a person who is interested in learning biology, and you want to give an overview of the most important things to know about that topic.

Topic 1: _______________

Topic 2: _______________

III. Skills. Open Notebook.

29. For each of the plants provided, make two observations. Based upon your observations, draw an inference about the climate where each of these plants would survive best.

Plant A	Plant B
Observations	
1.	1.
2.	2.
Inference	
1.	1.

30. On the graph paper provided, make a graph summarizing the average weights of the animals listed in the table below.

Animal Type	Number of Animals Weighed	Average Weight *
brush rabbit	13	2 kg
coyote	12	16 kg
kangaroo rat	27	160 g
bobcat	4	9 kg

* Note that not all weights are given in the same units. Be sure to label your graphs completely.

31. The tables below summarize the number of birds having beaks of various lengths in a population of finches.

Table A
Beak Size of Finches

Length of Beak	Percentage of Population
12	4
4	11
10	12
6	35
8	38

Table B
Beak Size of Finches

Length of Beak (mm)	Percentage of Population
4	11
6	35
8	38
10	12
12	4

Assume your fourth grade students made these tables. What improvement(s), if any, would you suggest they make

Table A -

Table B

32. Select the following fossils from the box: #10, Coral; #14, Brachiopod; #19, Ammonite; #22, Trilobite; and #30, Shark's tooth. Make a hierarchical classification scheme that fifth grade students could use to identify these five fossils. Use observable traits, not names or phyla. Don't use size because size can be quite variable.

33. Jeff did an experiment to determine whether grass seeds germinate better in the light or the dark. He planted 40 Fescue grass seeds on top of moist potting soil in each of two 3.5 oz. cups. Then he covered each container with an upside-down clear plastic cup so that water which evaporated would run back into the soil. He placed the 'Dark' sample in a cupboard and taped the door shut. He placed the 'Light' sample near an open window. The temperature outside was balmy most of the time, but 8 days after Jeff began the experiment, the temperature dropped to 33° F. for one night. Jeff obtained the following data:

	Number of Sprouts	
	Day 7	Day 14
Light	19	21
Dark	18	38

a. What proportion of the seeds germinated in each group?

Light ______ %　　　Dark _____ %

b. Explain the results.

c. Name two ways to improve the experiment so as to obtain more reliable results next time.
c.1)

c.2)

34. In what major ways is the life cycle of a mealworm different from that of a cricket?

35. Using information from your notebook, draw a timeline for the cricket reproductive cycle.

36. If your baby crickets survive, what is the earliest you would expect them to begin chirping? (approximate date) Explain your reasoning.